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# Portfolio Diversification, Market Power, and the Theory of the Firm

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## Abstract

This paper develops a model of firm behavior in the context of oligopoly and portfolio diversification by shareholders. Competition for shareholder votes among potential managers seeking corporate office leads to internalization and aggregation of shareholder objectives, including shareholdings in other firms, and the fact that shareholders are consumers and workers of the firms. When all shareholders hold market portfolios, firms that are formally separate behave as a single firm. I introduce new indices that capture the internalization effects from consumer/worker control, and discuss implications for antitrust, stakeholder theory, and the boundaries of the firm.

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Ownership of publicly traded firms by financial institutions has increased dramatically in recent decades. The fraction of U.S. stock market capitalization held by institutional investors increased from 7% in 1950 to about 70% today.<sup>1</sup> A natural consequence of this development is the ubiquity of common ownership of publicly traded firms by large financial institutions. The probability that two randomly selected firms in the same industry from the S&P 1500 have a common shareholder with at least 5% stakes in both firms increased from less than 20% in 1999Q4 to around 90% in 2014Q4 (Figure 1).<sup>2</sup> Thus, while there has been some degree of overlap for many decades, the current ubiquity of common ownership of large blocks of stock is a relatively recent phenomenon. This large increase in interlocking shareholdings coincided with the period of fastest growth in corporate profits and the fastest decline in the labor share since the end of World War II, with corporate profits approximately doubling as a share of GDP over the 10-year period between 2000 and 2010 (Figure 2).<sup>3</sup>

Changes of this magnitude in the ownership structure of corporations raise important questions about firm objectives and boundaries. To the extent that a group of formally separate firms have similar owners, should we continue to think of them as independent firms, or as part of a larger corporate structure? The answer to this question has significant consequences for antitrust policy, corporate governance, and for the distribution of income between capital and labor. Does common ownership lead firms in the direction of joint profit maximization? To what extent do firms internalize other shareholder objectives, such as shareholders being affected as consumers or as workers (and should they)? What role, if any, has the increase in common ownership played in the increasing share of corporate profits in GDP and the decline in the labor share?

Motivated by these questions, this paper develops a model of firm behavior in the context

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<sup>1</sup>See [Gompers and Metrick \(2001\)](#), [Gillan and Starks \(2007\)](#), [Davis \(2008\)](#), and [Blume and Keim \(2012\)](#).

<sup>2</sup>Figure 1 shows dramatic increases are also observed when using 3%, 4% 6%, or 7% thresholds for the ownership stakes instead of a 5% threshold.

<sup>3</sup>The inverse of the labor share is commonly used to proxy for aggregate markups in the macroeconomics literature ([Rotemberg and Woodford, 1999](#)) and [Nekarda and Ramey \(2010\)](#). Using a more sophisticated methodology, [Barkai \(2016\)](#) estimates markups for the U.S. economy and also finds that they have increased in recent decades.

of oligopoly/oligopsony and portfolio diversification by shareholders. In the model, based on a standard and tractable version of probabilistic voting, several potential managers are seeking to obtain the rents associated with corporate office, but they need to compete for shareholder votes in order to obtain these rents. This process of managerial competition and selection leads to internalization and aggregation of shareholder objectives, including the fact that they can hold shares in other firms, and the fact that shareholders are, to some extent, consumers and workers of the firms.

We start by applying the framework to prove a version of a “folk theorem:” the idea that, when there is no uncertainty and shareholders are not consumers or workers of the firms, if all shareholders hold market portfolios then all firms act as if they were a single firm to maximize joint profits (Rotemberg, 1984; Gordon, 1990; Hansen and Lott, 1996). In the case of certainty, there is no disagreement between shareholders. We then extend this result to the case of uncertainty—and therefore disagreement between shareholders—and show that under some conditions all firms still act as a single firm.

We then consider cases with general portfolios and where shareholders can be consumers or workers. In the case of oligopolistic firms with shareholder-consumers competing à la Cournot, we obtain an expression linking the average industry markup to the difference between an O’Brien and Salop (2000) modified Herfindahl-Hirschman Index (MHHI), and an index that captures the internalization effect from the fact that shareholders can also be consumers (which we name the “consumer internal power index” or “CIPI”). The CIPI captures the internal power of consumers within the firm associated with representation in corporate decisions, in contrast with external power of consumers from having access to competitor firms in the market.

In the case where ownership is proportional to consumption, then the internalization of the effect of market power on shareholders as consumers is complete, and the firm sets price equal to marginal cost (Nielsen, 1979; Farrell, 1985). While this case seems unrealistic for publicly traded firms, it could be a good approximation (as well as a rationale) for consumer cooperatives or publicly owned firms. We then derive an expression for the consumer internal power

index in terms of the variances of consumption and ownership shares of the firm, as well as the relative alignment of each these with control of the firm (relative alignment is measured by the ratio of the cosine of the angle between the consumption and control share vectors and the cosine of the ownership and control share vectors).

We next consider the case of labor market oligopsony, again with firms competing à la Cournot, the model yields an expression linking the average industry markdown of wages below the value of the marginal product of labor (weighted by market shares) to the difference between an MHHI based on the employment shares of the firms, and an index that captures the internalization effect from the fact that shareholders can also be workers (which we name the “worker internal power index” or “WIPI”).<sup>4</sup> As with the CIPI, the WIPI captures the internal power of the workers within the firm that associated with representation in corporate decisions, in contrast with the external power of workers from having access to competitor firms in the market.

The wage gets closer to the marginal product of labor as the shares of labor supply and ownership become more closely aligned. The worker internal power index would tend be higher for worker cooperatives and at large firms in countries where some level of worker control is mandated by law. We also show that there is a level of legally mandated worker control that would restore efficiency in the presence of market power in the labor market. However, in general the optimal level depends not just on firm size but on the whole ownership structure.

The main takeaway from the analysis of labor market oligopsony with shareholder voting is that who owns what matters. This is for two reasons. First, it matters because the level of common ownership partly determines the level of market power in a labor market. And second, it matters because the degree to which the workers are owners of the firms determines the extent to which market power is used to reduce wages below the value of the marginal

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<sup>4</sup>Labor economics has studied monopsony power in the labor market since at least [Robinson \(1933\)](#). [Manning \(2003\)](#) provides a great overview and exposition of this literature. Relative to the existing literature, the present paper focuses on the case of Cournot oligopsony and the relation between the markdown and labor market concentration, and on the effect of worker ownership and control.

product of labor and the extraction of monopsony rents from the workers.

The model can thus shed some light on the debates on shareholder value maximization, stakeholder representation, and corporate social responsibility at large firms. [Friedman \(1970\)](#) argued that firms should focus solely on individual profit maximization, and [Jensen \(2001\)](#), somewhat more generally, argued that a firm should maximize its market value, and dismissed stakeholder representation as infeasible. [Magill, Quinzii, and Rochet \(2013\)](#), on the other hand, argue that stakeholder representation is feasible as long as stakeholder groups and their objectives are well defined. The theory developed in the present paper provides a clear efficiency rationale for some degree of consumer and employee representation at large firms as one possible response to market power in product and labor markets, and perhaps as a complement to antitrust policy, since it seems unlikely that antitrust can deal effectively with all the instances of market power in the labor market. The reason is that consumer and employee representation can reduce the markdown of wages relative to the marginal product of labor and therefore bring the economy closer to a competitive outcome.

This line of reasoning provides an efficiency rationale for reducing wealth inequality: to the extent that reducing inequality makes control, ownership, consumption, and labor supply more aligned, less inequality of ownership and control can automatically lead to increased representation of the interests of consumers and employees, as well as other social interests, in the objective function of the firm. In the limit, when agents are homogeneous and all firms are commonly owned, this automatic stakeholder representation leads to a Pareto efficient outcome even in the presence of market power, and even though there is no competition in the economy. Similarly—to the extent that there is “home bias” in consumption and labor supply—it provides an efficiency rationale for local ownership, since in this case local ownership would tend to align control of the firms with consumption and labor supply.

The fact that firms are connected through common ownership created by portfolio diversification also has important implications for the theory of the boundaries of the firm. Classic contributions to this theory include [Coase \(1937\)](#), [Williamson \(1975\)](#), [Grossman and Hart \(1986\)](#),

and [Hart and Moore \(1990\)](#). According to the Grossman-Hart-Moore theory, “a firm is exactly a set of assets under common ownership” ([Holmström and Roberts, 1998](#)). If every agent holds a market portfolio, then the ownership of all firms becomes identical, and therefore according to this definition they are really a single firm. This literature tends to focus on mergers and acquisitions as the source of changes in firm boundaries, and on the deep economic and strategic considerations that motivate mergers. In this paper, in contrast, we place the focus on portfolio diversification as a powerful force that determines ultimate ownership of firms by financial investors in stock market economies, with the effect of blurring firm boundaries at a massive scale, and in a way that is not driven by economic or strategic considerations beyond the management of portfolio risk. Another difference with this literature is that the focus in this paper is on market power, as opposed to the focus of the literature on hold-up problems and other contractual externalities.

The theory of the firm developed in this paper could also provide a solid foundation for models of general equilibrium oligopoly. A major issue that this literature faced was that the solution to the models depended on the choice of the numeraire ([Gabszewicz and Vial, 1972](#); [Roberts and Sonnenschein, 1977](#); [Mas-Colell, 1982](#); [Hart, 1982b,a](#); [Neary, 2002, 2009](#)). However, the reason for this problem was that the literature generally assumed that firms maximized profits, which are necessarily dependent on the unit of account. In contrast, when the objective of the firm is microfounded as the outcome of a voting process, it is independent of the numeraire. [Azar and Vives \(2016\)](#) build on this idea to develop a simple macroeconomic model to explore theoretically the effect of market concentration on aggregate outcomes.

## 1 Literature Review

Modern Portfolio Theory, as developed by [Markowitz \(1952\)](#), [Arrow \(1964\)](#), [Debreu \(1959\)](#), [Sharpe \(1964\)](#), and [Lintner \(1965\)](#) among others, prescribes that investors should hold market portfolios and thus provided intellectual support to the rise of mutual funds and index funds.



This body of work focused on the decision problem of an investor and its normative insights are from an individual investor's point of view. From the narrow point of view of portfolio diversification by investors, the rise of mutual funds and especially low-cost index funds has been a welcome development. However, the analysis of the implications for society as a whole was limited, for example, by the fact that the theory ignored the possibility of any effect on competition because this set of models assumed that firms were price-takers. An important but under-appreciated result in economic theory is that, when all shareholders hold market portfolios, are not consumers or workers of the firms, and there is no uncertainty, then they unanimously agree on joint profit maximization by all firms in the economy, leading to an economy-wide monopoly outcome (Rotemberg, 1984; Gordon, 1990; Hansen and Lott, 1996). Modern portfolio theory recommended that investors hold market portfolios, but had nothing to say about the competitive implications of everyone following its advice because any competitive implications were excluded by the price-taking assumption.

While the limiting result of economy-wide monopoly is useful to understand the potential consequences of the trend of increasing portfolio diversification, in practice many shareholders are still far from holding completely diversified portfolios. Shareholders also differ in other dimensions, such as the extent to which they are consumers and workers of the firms that they own, and in their levels of risk-aversion. Given that shareholders will often disagree, can a reasonable objective function of the firm be derived from aggregation of the individual objectives of heterogeneous shareholders? The answer is no, or at least not in a completely general way, as Kenneth Arrow discovered when, as a graduate student, he attempted to generalize the theory of the firm to the case of multiple shareholders (Arrow, 1983, p. 2):

“When in 1946 I began a grandiose and abortive dissertation aimed at improving on John Hicks's *Value and Capital*, one of the obvious needs for generalization was the theory of the firm. What if it had many owners, instead of the single owner postulated by Hicks? To be sure, it could be assumed that all were seeking to maximize profits; but suppose they had different expectations of the future? They would

then have differing preferences over investment projects. I first supposed that they would decide, as the legal framework would imply, by majority voting. In economic analysis we usually have many (in fact, infinitely many) alternative possible plans, so that transitivity quickly became a significant question. It was immediately clear that majority voting did not necessarily lead to an ordering. In view of my aims, I reacted to this result as a nuisance, not a clue for further study. Besides, I was convinced that what we presently call the Condorcet paradox was not new. I am at a loss to identify the source of my belief, now that I know the previous literature, since I could not possibly have seen any of this obscure material prior to 1946.”

Given this difficulty in dealing with the shareholder preference aggregation problem, economists chose to focus on cases in which shareholders unanimously agreed on market value maximization, which is the case when firms are price-takers and markets are complete (Ekern and Wilson, 1974; Radner, 1974; Leland, 1974). Hart (1979) proves a similar result when firms are competitive but markets are incomplete: shareholders agree on value maximization but disagree on how to achieve this goal, since not all prices are quoted in the market. Hart (1979) explicitly notes that in general shareholders may not want value-maximizing behavior because they could hold shares in other firms, and because they can be consumers:

“Without such a competitiveness assumption, a change in a firm’s production plan will have two effects on an initial shareholder’s utility. First there will be a change in the shareholder’s initial wealth due to changes in the (net) market value of the firm and possibly also in the (net) market values of other firms; we call this the wealth effect. Secondly, there will be changes in the prices of the bundles of contingent commodities purchased by the shareholder in the stock market; we call this the consumption effect.”

While other papers in this literature assume that firms are price-takers, Hart (1979) instead considers a sequence of replica economies and derives the value maximization result by im-

posing enough assumptions on agents' portfolios to prevent, among others, cases such as all shareholders holding market portfolios.

In contrast to this literature, we focus on the oligopoly case (or oligopsony), and develop a model of the firm in which shareholder preferences can be aggregated. This requires imposing more structure on shareholder preferences and the voting framework than Arrow did. In particular, I use the standard theory of probabilistic voting which was developed by [Lindbeck and Weibull \(1987\)](#) and [Coughlin \(1992\)](#) as a model of political competition.<sup>5</sup>

## 2 The basic framework: oligopoly with shareholder voting

An oligopolistic industry consists of  $N$  firms. Firm  $n$ 's profits per share can be random and depend both on its own strategies  $\mathbf{p}_n$  and on the strategies of the other firms,  $\mathbf{p}_{-n}$ , as well as the state of nature  $\omega \in \Omega$ :

$$\pi_n = \pi_n(\mathbf{p}_n, \mathbf{p}_{-n}; \omega).$$

Suppose that  $\mathbf{p}_n \in S_n \subseteq \mathbb{R}^K$ , so that strategies can be multidimensional. The strategies of the firm can be prices, quantities, investment decisions, innovation, or in general any decision variable that the firm needs to choose. In principle, the strategies could be contingent on the state of nature, but this is not necessary.

There is a set  $I$  of shareholders, whose portfolios can include shares in more than one firm in the industry. Shareholder  $i$  holds  $\beta_{in}$  shares in firm  $n$ . The total number of shares of each firm is normalized to 1. For simplicity, we assume one vote per share, although this can easily be relaxed. Each firm holds its own elections to choose its management, which controls the firms' strategies. At each company  $n$ , there are two potential managers,  $A_n$  and  $B_n$ , in Downsian competition for the shareholder's votes. Let  $\xi_{i,J_n}$  denote the probability that shareholder  $i$  votes for managerial candidate  $J_n$  in company  $n$ 's elections, where  $J_n \in \{A_n, B_n\}$ . The expected vote

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<sup>5</sup>Some papers in the incomplete markets literature also used voting models to aggregate shareholder preferences. For example, median voter theory is used by [Benninga and Muller \(1979\)](#) and [DeMarzo \(1993\)](#).

share of candidate  $J_n$  in firm  $n$ 's elections is

$$\tilde{\xi}_{J_n} = \sum_{i \in I} \beta_{in} \tilde{\xi}_{i,J_n}.$$

Shareholders get utility from income, which is the sum of profits from all their shares. I will also assume that utility has a random component that depends on what candidate is in power in each of the firms.<sup>6</sup> One way to think about the random component of utility is that managerial candidates cannot predict perfectly the way in which a particular shareholder will vote in the election. That is, there is some uncertainty about shareholder voting behavior from the point of view of the managerial candidates. More activist shareholders will communicate with potential managers to reduce this uncertainty. As we will see, the higher the level of information the managerial candidates have about a given shareholder's preferences, the higher that shareholder's weight is in the firm's equilibrium objective function.

The utility of shareholder  $i$  when the strategy of firm  $n$  is  $\mathbf{p}_n$ , the strategies of the other firms are  $\mathbf{p}_{-n}$ , and the vector of elected parties is  $\{J_n\}_{n=1}^N$  is

$$\tilde{U}_i \left( \mathbf{p}_n, \mathbf{p}_{-n}, \{J_s\}_{s=1}^N \right) = U_i(\mathbf{p}_n, \mathbf{p}_{-n}) + \sum_{s=1}^N \tilde{\sigma}_{is}(J_s),$$

where  $U_i(\mathbf{p}_n, \mathbf{p}_{-n}) = \mathbb{E} \left[ u_i \left( \sum_{s=1}^N \beta_{is} \pi_s(\mathbf{p}_s, \mathbf{p}_{-s}; \omega) \right) \right]$ . The utility function  $u_i$  of each group is increasing in income, with non-increasing marginal utility. The  $\tilde{\sigma}_{in}(J_n)$  terms represent the random utility that shareholder  $g$  obtains if candidate  $J_i$  controls the board of company  $n$ . The random utility terms are independent across firms and shareholders, and independent of the state of nature  $\omega$ . As a normalization, let  $\tilde{\sigma}_{in}(A_n) = 0$ . To ensure existence of equilibrium in this relatively general case, we make the following assumptions:

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<sup>6</sup>Alternatively, one could consider a deterministic voting model, which would lead to firms in equilibrium maximizing a weighted-median of shareholder utilities. The intuition about internalization of overlapping ownership stakes and of the effects of firm prices and wages on shareholders as consumers and workers would be the same. A probabilistic voting is more tractable than a deterministic model, and arguably more realistic, given that in practice managers are unlikely to be completely certain about which way a particular shareholder will vote in an election.

**Assumption 1.** (Compact and convex strategy spaces) *The strategy spaces  $S_n$  are nonempty compact convex subsets of Euclidean space.*

**Assumption 2.** (Continuous shareholder utilities) *The deterministic component of utility,  $U_i(\mathbf{p}_n, \mathbf{p}_{-n})$ , is continuous and twice differentiable in  $(\mathbf{p}_n, \mathbf{p}_{-n})$  and has continuous second derivatives.*

**Assumption 3.** (Concave shareholder utilities) *The deterministic component of utility,  $U_i(\mathbf{p}_n, \mathbf{p}_{-n})$ , is strictly concave in  $\mathbf{p}_n$ .*

**Assumption 4.** (Uniform distribution of bias) *The bias of voter  $i$  for candidate  $B_n$ ,  $\tilde{\sigma}_{in}(B_n)$  is distributed uniformly on  $\left[-\frac{1}{2\psi_{in}}, \frac{1}{2\psi_{in}}\right]$ .*

While these assumptions are fairly restrictive, especially concavity of shareholder utilities, they are standard in the probabilistic voting literature and used to ensure existence of an equilibrium. They are sufficient but not necessary, and can be relaxed in particular applications of the theory. For example, in later sections in which we apply the framework in a Cournot context we relax concavity of individual shareholder objective functions and assume a weaker condition.

Let  $\mathbf{p}_{A_n}$  denote the platform of candidate  $A_n$  and  $\mathbf{p}_{B_n}$  that of candidate  $B_n$ .

**Assumption 5.** (Conditional sincerity) *Voters are conditionally sincere. That is, in each firm's election they vote for the candidate whose strategies maximize their utilities, given the equilibrium strategies in all the other firms. In case of indifference between the two parties, a voter randomizes.*

Conditional sincerity ([Alesina and Rosenthal, 1995, 1996](#); [Chari, Jones, and Marimon, 1997](#); [Ahn and Oliveros, 2012](#)) is a natural assumption as a starting point in models of multiple elections. [Alesina and Rosenthal \(1996\)](#) obtain it as a result of coalition proof Nash equilibrium in a model of simultaneous presidential and congressional split-ticket elections.<sup>7</sup>

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<sup>7</sup>In this paper, I treat conditional sincerity as a plausible behavioral assumption, which, while natural as a starting point, does not necessarily hold in general. A complete characterization of the conditions under which conditional sincerity arises as the outcome of strategic voting in models of multiple elections is an open problem.

Using Assumption 5, the probability that shareholder  $i$  votes for candidate  $A_n$  is

$$\xi_{i,A_n} = P[\tilde{\sigma}_{in}(B_n) < U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n}) - U_i(\mathbf{p}_{B_n}, \mathbf{p}_{-n})].$$

As notation, we call  $H_{in}$  the cumulative distribution function of  $\tilde{\sigma}_{in}(B_n)$  (since it is uniform,  $H_{in}(x) = \frac{1}{2} + \psi_{in}x$ ). The vote share of candidate  $A_n$  is

$$\zeta_{A_n} = \sum_{i \in I} \beta_{in} H_{in} [U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n}) - U_i(\mathbf{p}_{B_n}, \mathbf{p}_{-n})].$$

We assume that candidates choose their platforms to maximize their expected vote shares:

**Assumption 6.** (Vote share objective) *Candidates maximize expected vote share at their firm.*

Elections for all companies are held simultaneously, and the two parties in each company announce their platforms simultaneously as well. A pure-strategy Nash equilibrium for the industry under vote share objective is a set of platforms  $\{\mathbf{p}_{A_n}, \mathbf{p}_{B_n}\}_{n=1}^N$  such that, given the platform of the other candidate in the firm, and the winning strategies in all the other firms, a managerial candidate chooses its platform to maximize its vote share. The first-order condition for managerial candidate  $A_n$  is

$$\sum_{i \in I} \psi_{in} \beta_{in} \frac{\partial U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n})}{\partial \mathbf{p}_{A_n}} = 0,$$

where

$$\frac{\partial U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n})}{\partial \mathbf{p}_{A_n}} = \left( \frac{\partial U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n})}{\partial p_{A_n}^1}, \dots, \frac{\partial U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n})}{\partial p_{A_n}^K} \right).$$

In the latter expression,  $p_{A_n}^k$  is the  $k$ th component of the strategy vector  $\mathbf{p}_{A_n}$ . The derivatives in terms of the profit functions are

$$\frac{\partial U_i(\mathbf{p}_{A_n}, \mathbf{p}_{-n})}{\partial \mathbf{p}_{A_n}} = \mathbb{E} \left[ u'_i \left( \sum_{s=1}^N \beta_{is} \pi_s(\mathbf{p}_s, \mathbf{p}_{-s}; \omega) \right) \sum_{s=1}^N \beta_{is} \frac{\partial \pi_s(\mathbf{p}_s, \mathbf{p}_{-s})}{\partial \mathbf{p}_{A_n}} \right].$$

The maximization problem for managerial candidate  $B_n$  is symmetric.

**Proposition 1.** *Under assumptions 1, 2, 3, 4, 5, and 6, a pure-strategy equilibrium of the voting game exists. In the equilibrium, the two candidates at every firm converge to the same strategy, which maximizes a weighted average of shareholder utilities conditional on rival strategies:*

$$\sum_{i \in I} \psi_{in} \beta_{in} U_i(\mathbf{p}_n, \mathbf{p}_{-n}) \text{ for } n = 1, \dots, N.$$

*Proof.* In Appendix. ■

□

Note that the weights in the average of shareholder utilities are different at different firms. The maximization takes into account the effect of the strategies of firm  $n$  on the profits that shareholders get from every firm, not just firm  $n$ . Thus, when the owners of a firm are also the residual claimants for other firms, they internalize some of the pecuniary externalities that the actions of the first firm generate for the other firms that they hold.

Alternatively, one could assume that managerial candidates maximize the probability of winning the election. This would be the case if there is, say, an exogenous rent from corporate office, and managerial candidates maximize the expected utility from that rent, which is the constant rent times the probability of obtaining it. If the candidates maximize win probability, then a similar characterization of the objective function of the firm obtains, but with Banzhaf voting power indices, defined as the probability that a voter (in this case a shareholder) is pivotal in an election with two options, if the other voters vote randomly for each option with equal probabilities, taking the place of the voting shares in the weights of the firms' objective functions.<sup>8</sup> The Banzhaf index tends to assign more than proportional weight to large shareholders, since they are more likely to be pivotal than smaller shareholders. As a shareholders shares approach 50%, the probability of being pivotal approaches 100%, and thus the shareholder gets close to complete control of the firm.

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<sup>8</sup>This is sometimes also referred to as the Penrose-Banzhaf index. See [Penrose \(1946\)](#) and [Banzhaf III \(1964\)](#).

It would also be possible to add a previous stage in the model in which shareholders can invest in increasing their control shares of the firm, either by paying a cost of information to reduce the uncertainty in their voting behavior (Matějka and Tabellini, 2016), or by buying votes directly, if this were institutionally possible (Dekel and Wolinsky, 2012; Posner and Weyl, 2013).<sup>9</sup> This would further increase the power of larger shareholders relative to small shareholders. Small shareholders would have little incentive to pay the costs, since their stakes in the firm are small. Large shareholders would have a relatively higher incentive to pay the cost of activism, given their higher stakes.

Thus, while direct communication between shareholders and managers is not necessary for managers to internalize shareholder interests in their decision-making, a particular shareholder investing in information and engaging with management to communicate its objectives could achieve a higher weight in the objective of the firm relative to other shareholders. If large shareholders have portfolios that are markedly different from those of smaller shareholders, the increased representation of large shareholders could be to the detriment of small shareholders.

### **3 A folk theorem: complete diversification of shareholder portfolios leads to monopoly**

As a first application of the voting model, in this section we use this framework to prove formally a version of a “folk theorem:” the idea that when all shareholders hold market portfolios, then all firms in an economy act as a single firm, because shareholders agree unanimously on joint profit maximization (Rotemberg, 1984; Gordon, 1990; Hansen and Lott, 1996). This shareholder unanimity result also requires that shareholders are not consumers or suppliers of factors of production to the firms, and that they are not affected by non-pecuniary externalities, which we have assumed in the development of the oligopoly with shareholder voting model,

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<sup>9</sup>In a practice called “empty voting”, hedge funds can construct financial instruments to replicate a voting share without financial interest. For more details, see, for example, Hu (2015).



but which could easily be relaxed. We then extend this result to the case in which all shareholders hold market portfolios but potentially disagree on firm strategies due to uncertainty. It would be straightforward to generalize this result to cases in which shareholders disagree for other reasons, including cases where shareholders are consumers or suppliers, or are affected by non-pecuniary externalities.

We will find it useful to define complete diversification:

**Definition 1.** (*Complete Diversification*) *A shareholder who holds a market portfolio, that is any portfolio that is proportional to the total number of shares of each firm, is completely diversified.*

**Theorem 1** (Folk). *Suppose all shareholders are completely diversified, and there is no uncertainty. Then the equilibrium of the voting game yields the same outcome as the one that a monopolist who owned all the firms and maximized their joint profits would choose.*

*Proof.* In Appendix. ■

□

The intuition is simple: when all shareholders are completely diversified and there is no uncertainty, then there is no conflict of interest among them, and the aggregation problem becomes trivial. Thus, in the special case of complete diversification and certainty (or risk-neutrality), shareholders would be unanimous in their support for joint profit maximization as the objective of the firm. In fact, any preference aggregation method satisfying Arrow's unanimity axiom would yield the same outcome.

For the more general case with shareholder disagreement due to uncertainty (and allowing for risk-averse shareholders), we can show a similar result. We need an additional assumption, which is that the distribution of the bias be the same across firms for a given shareholder, even when the distribution can be different for different shareholders. In terms of the uniform distribution, this requires that the parameter  $\psi_{in}$  depend only on  $i$  and not on  $n$ :

**Assumption 7.** (Same distribution of bias across firms for a given shareholder) *The of the distribution of the bias terms depends on  $i$  but not on  $n$ . That is,  $\psi_{in} = \psi_i$  for all  $n$  and all  $i$ .*

**Theorem 2.** *Suppose all shareholders are completely diversified, and assumption 7 holds. Then the equilibrium of the voting game yields the same outcome as the one that a monopolist who owned all the firms and maximized a weighted average of the utilities of the shareholders would choose.*

*Proof.* In Appendix. ■

□

Note that, although all the shareholders hold proportional portfolios, there is still a conflict of interest between them. This is due to the fact that there is uncertainty and shareholders, unless they are risk neutral, care about the distribution of joint profits, not just the expected value. For example, they may have different degrees of risk aversion, both because some may be wealthier than others (i.e. hold a bigger share of the market portfolio), or because their utility functions differ. Although all shareholders are fully internalizing the pecuniary externalities that the actions of each firm generates on the profits of the other firms, some may want the firms to take on more risks, and some may want less risky actions. Thus, in this case there is still a non-trivial preference aggregation problem. The assumption that the distribution of bias is the same across firms for a given shareholder (even when it can differ across shareholders), ensures that in equilibrium each firm is maximizing same objective function. In other words, in a sense all firms are aggregating shareholder preferences in the same way, and as a result they act as if they were a single firm.

As we already noted, an almost identical result, with the same logical structure for the proof, would apply to cases in which the shareholders disagree for other reasons, such as price effects on shareholders as consumers, workers, and suppliers of factors of production more generally, or non-pecuniary externalities generated by the firms. In those cases a monopolist that maximized a weighted average of the utilities of its shareholders would internalize, to some extent, their interests as consumers and factor suppliers and the externalities, and an equivalent outcome would be obtained in an oligopoly with completely diversified shareholders.

In the next sections, we consider in more detail cases in which portfolios are incompletely diversified, and quantify explicitly the internalization of consumption and labor supply effects.

## 4 Oligopoly and shareholder-consumers

In this section, we relax the assumption that the shareholders are not consumers of the firms. Consider the case in which a continuum  $I$  of shareholder-consumers of measure 1, and  $N$  oligopolistic firms producing a good  $x$  with price  $p$  and competing in quantities. Shareholder  $i$  owns a fraction  $\beta_{ij}$  of the shares firm  $j$ , with  $\int_{i \in I} \beta_{ij} = 1$  for all  $j$ . There is another good  $y$ , which we call money and serves as the numeraire. Shareholder  $i$  derives utility from consuming good  $x$  and from money:

$$U_i(x_i, y_i) = u_i(x_i) + y_i,$$

with  $u'_i > 0$  and  $u''_i < 0$ . The total wealth  $m_i$  of shareholder  $i$  is the sum of initial monetary wealth  $m_{i,0}$  plus the value of shares in the firms, leading to the budget constraint

$$px_i + y_i \leq m_{i,0} + \sum_{j=1}^N \beta_{ij} \pi_j,$$

where  $\pi_j$  are the total profits of firm  $j$ , which the shareholders, being atomistic, take as given in their consumption decisions.

Assuming an interior solution, the first-order conditions lead to the usual expression for the demand for good  $x_i$  (assuming that the oligopolistic industry is small enough relative to the economy to have an interior solution):

$$u'_i(x_i) = p.$$

Shareholder  $i$  has indirect utility function  $v_i(p, m_i)$ . Shareholder  $i$ 's demand for good  $x$  is  $x_i(p) = (u'_i)^{-1}(p)$ . Aggregate demand for good  $x$  is

$$D(p) = \int_{i \in I} x_i(p) di = \int_{i \in I} (u'_i)^{-1}(p) di.$$

We will find it useful to express this relationship in terms of the inverse demand function  $p(Q)$ , where  $Q$  is the aggregate quantity produced in the industry.

Firm  $j$ 's profits are given by  $\pi_j = p(\sum_{k=1}^N q_k)q_j - c_j(q_j)$  with  $c_j(q_j)$  being its total cost function, which we take as exogenous, since we are assuming that firms are price-takers in factor markets. The managerial candidates of firm  $j$  choose strategies to maximize a weighted average of the utilities of the shareholders, with weights proportional to the number of shares times the density of their bias:

$$\max_{q_j} \int_{i \in I} \gamma_{ij} v_i(p(q_j + q_{-j}), m_i(q_j; q_{-j})) di,$$

where  $\gamma_{ij}$  is equal to  $\frac{\beta_{ij}\psi_{ij}}{\int_{s \in I} \beta_{sj}\psi_{sj} ds}$ , and can be thought of as shareholder  $i$ 's control share at firm  $j$ .

The first-order condition for a managerial candidate at firm  $j$  is:

$$\int_{i \in I} \gamma_{ij} \left\{ \frac{\partial v_i}{\partial p} p'(Q) + \frac{\partial v_i}{\partial m_i} \left( \sum_k \beta_{ik} p'(Q) q_k + \beta_{ij} (p(Q) - c'_j(q_j)) \right) \right\} di = 0.$$

Using Roy's identity, we can rewrite this as

$$\int_{i \in I} \gamma_{ij} \left\{ -\frac{\partial v_i}{\partial m_i} \left( p'(Q) x_i(p(Q)) + \sum_k \beta_{ik} p'(Q) q_k + \beta_{ij} (p(Q) - c'_j(q_j)) \right) \right\} di = 0.$$

Since the marginal utility of wealth is equal to 1 for all shareholders due to the quasilinearity assumption, we can rewrite this as

$$\int_{i \in I} \gamma_{ij} \left\{ p'(Q) \left( \sum_k \beta_{ik} q_k - x_i(p(Q)) \right) + \beta_{ij} (p(Q) - c'_j(q_j)) \right\} di = 0.$$

A sufficient condition to ensure strict concavity of the individual shareholders' objective functions with respect to the quantity of firm  $j$  is:

$$p''(Q) \left( \sum_k \beta_{ik} q_k - x_i(p(Q)) \right) + p'(Q) \left( \beta_{ij} - \frac{\partial x_i}{\partial p} p'(Q) \right) + \beta_{ij} (p'(Q) - c''_j(q_j)) < 0.$$

This condition ensures the existence of equilibrium in the Cournot model with shareholder-consumers.<sup>10</sup>

We can then use the first-order conditions to characterize the equilibrium as follows:

**Proposition 2.** *In a model of Cournot oligopoly with shareholder-consumers, the share-weighted average markup in equilibrium is:*

$$\sum_j s_j \frac{p - c'_j}{p} = \frac{1}{\eta} (\text{MHHI} - \text{CIPI}),$$

where  $\eta$  is the elasticity of demand, the MHHI is the modified Herfindahl-Hirschman Index, defined as

$$\text{MHHI} \equiv \sum_j \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j s_k,$$

where  $\eta$  is the elasticity of demand for  $x$  and  $s_j$  is the market share of firm  $j$  in the market for  $x$ ; and the CIPI is the consumer internal power index, defined as:

$$\text{CIPI} \equiv \sum_j \frac{\int_{i \in I} \gamma_{ij} \mu_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j,$$

where  $\mu_i$  is the fraction of the product consumed by shareholder  $i$ .

*Proof.* In Appendix. ■

□

The consumer internal power index quantifies the alignment between shareholders' consumption and control vectors. By rewriting the inner products in terms of the corresponding cosine similarities<sup>11</sup> and the norms, we obtain the following result:

<sup>10</sup>A weaker condition is assuming that this holds for the average shareholder (with the average calculated using weights  $\gamma_{ij}$ ):

$$\int_{i \in I} \gamma_{ij} \left\{ p''(Q) \left( \sum_k \beta_{ik} q_k - x_i(p(Q)) \right) + p'(Q) \left( \beta_{ij} - \frac{\partial x_i}{\partial p} p'(Q) \right) + \beta_{ij} \left( p'(Q) - c'_j(q_j) \right) \right\} di < 0.$$

<sup>11</sup>The cosine similarity between two vectors is defined as the cosine of the angle between the two vectors, and

**Proposition 3.** *The CIPI can be expressed in terms of the variance of consumption shares, the variances of the ownership shares of the various firms, and the ratios of cosine similarities between consumption and control shares and between ownership and control shares:*

$$CIPI = \sum_j \frac{\cos(\alpha_{\gamma_j, \mu}) \sqrt{\sigma_{\mu}^2 + 1}}{\cos(\alpha_{\gamma_j, \beta_j}) \sqrt{\sigma_{\beta_j}^2 + 1}} s_j,$$

where  $\alpha_{\gamma_j, \mu}$  is the angle between the consumption share vector  $\mu$  and the control share vector of firm  $j$ ,  $\gamma_j$ ,  $\alpha_{\gamma_j, \beta_j}$  is the angle between the ownership share vector and the control share vector of firm  $j$ , and  $\sigma_{\mu}^2$  and  $\sigma_{\beta_j}^2$  are the variances of the consumption share and ownership share vectors, respectively.

*Proof.* In Appendix. ■ □

A look at this formula suggests that consumption internalization is unlikely to be large in most applications, because empirical estimates of the variance of the wealth distribution in most economies indicate that the variance is very large, if not infinite.<sup>12</sup> For example, [Vermeulen \(2014\)](#) estimates the parameters of wealth inequality for the U.S. and several European countries under a Pareto parameterization, and finds that the shape parameter estimates are centered between 1 and 2 for all the countries considered (the Pareto distribution has an infinite variance when the shape parameter is between 1 and 2). Wealth in the form of stock-ownership is even more highly concentrated than wealth in general ([Wolff, 2010](#)), although obviously this may not apply to all firms, for example small locally-owned family firms. The distribution of consumption, on the other hand, is close to log-normal ([Battistin, Blundell, and Lewbel, 2009](#)), and thus has finite variance.

The analysis in this section also provides an efficiency justification for policies to reduce wealth inequality, to the extent that less inequality of ownership and control leads to more representation of stakeholder and social interests in the objective function of the firm. The latter can be interpreted as an uncentered measure of correlation between the two vectors. Since it is a cosine, its value is between  $-1$  and  $1$ .

<sup>12</sup>Or, to be more precise, that the wealth distribution in the actual population is estimated to be drawn from a distribution with infinite variance, since the draws cannot have infinite variance themselves.

leads to internalization of externalities that firms generate on stakeholders as in [Magill, Quinzii, and Rochet \(2013\)](#). Similar arguments would apply to pollution externalities. This analysis also points to a possible benefit of local ownership: to the extent that there is “home bias” in consumption or in other externalities, local ownership can make ownership more aligned with stakeholder interests.

## 5 Labor-market oligopsony and shareholder-workers

Consider a small industry that sells its output in the world market, taking the price as given. There are  $N$  firms that have market power in the local labor market. Firm  $j$  has a production function that uses only labor  $F_j(L_j)$  (capital or land can be thought of as fixed factors). There is a continuum of shareholders of the firms of measure 1, who also supply labor. Each shareholder has a time endowment  $T$ . Their utility functions for leisure  $l$ , which has price  $w$  and money  $y$ , which is the numeraire, are quasilinear:

$$U_i(l_i, y_i) = u_i(l_i) + y_i,$$

with  $u'_i > 0$  and  $u''_i < 0$ . The total wealth  $m_i$  of shareholder  $i$  is  $m_i$ , equal to initial monetary wealth  $m_{i,0}$ , plus labor income from selling their time endowment, plus the value of their ownership in the oligopsonistic firms. The budget constraint of shareholder  $i$  is

$$wl_i + y_i \leq m_{i,0} + wT + \sum_{j=1}^N \beta_{ij} \pi_j.$$

Assuming an interior solution, the first-order conditions lead to the usual expression for the demand for leisure  $l_i$ :

$$u'_i(l_i) = w.$$

Shareholder  $i$  has indirect utility function  $v_i(w, m_i)$ .

The labor supply of shareholder  $i$  is  $L_i(w) = T - l_i(w)$ . Aggregate labor supply is

$$L(w) = \int_{i \in I} L_i(w) di = T - \int_{i \in I} l_i di = T - \int_{i \in I} (u'_i)^{-1}(w) di.$$

As before, we will find it useful to express it in terms of an inverse labor supply function  $w(L)$ , where  $L$  is the total amount of labor demanded by the firms.

Firm  $j$ 's profits are given by  $\pi_j = pF_j(L_j^d) - w(L_j^d, L_{-j}^d)L_j^d$ . Each managerial candidate at each firm maximizes a weighted average of shareholder-worker utilities, with weights given by the control shares in the firm  $\gamma_{ij}$ :

$$\max_{L_j^d} \int_{i \in I} \gamma_{ij} v_i \left( w(L_j^d; L_{-j}^d), m_i(L_j^d; L_{-j}^d) \right) di,$$

where  $v_i(w, m_i)$  is the indirect utility function of shareholder  $i$ , and wealth is  $m_i(L_j, L_{-j}) \equiv m_{i,0} + w(L_j^d; L_{-j}^d)T + \sum_k \beta_{ik} \pi_k(L_j^d; L_{-j}^d)$ . Taking first-order conditions with respect to the labor demand of firm  $j$  yields:

$$\int_{i \in I} \gamma_{ij} \left\{ \frac{\partial v_i}{\partial w} w'(L) + \frac{\partial v_i}{\partial m_i} \left( w'(L)T - \sum_k \beta_{ik} w'(L)L_k^d + \beta_{ij} (pF'_j(L_j^d) - w(L)) \right) \right\} di = 0.$$

Using Roy's identity, and the fact that the marginal utility of wealth is constant, we can rewrite this as:

$$\int_{i \in I} \gamma_{ij} \left\{ w'(L) \left( L_i(w(L)) - \sum_k \beta_{ik} L_k^d \right) + \beta_{ij} (pF'_j(L_j^d) - w(L)) \right\} di = 0.$$

A sufficient condition for the objective function of each shareholder to be strictly concave is

$$w''(L) \left( L_i(w(L)) - \sum_k \beta_{ik} L_k^d \right) + w'(L) \left( \frac{\partial L_i}{\partial w} w'(L) - \beta_{ij} \right) + \beta_{ij} (pF''_j(L_j^d) - w'(L)) < 0$$

As before, we can weaken this condition by requiring only that it holds for the weighted-



average shareholder. We can then characterize the equilibrium in a way analogous to that for the case of oligopoly with shareholder consumers:

**Proposition 4.** *In a model of Cournot labor-market oligopsony with shareholder-workers, the share-weighted average markup in equilibrium is:*

$$\sum_j s_j^L \frac{F_j' - \frac{w}{p}}{\frac{w}{p}} = \frac{1}{\eta^L} (\text{MHHI} - \text{WIPI}),$$

where  $\eta$  is the elasticity of labor supply, the MHHI is the modified Herfindahl-Hirschman Index calculated using the firms' shares of employment, defined as

$$\text{MHHI} \equiv \sum_j \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j^L s_k^L,$$

where  $s_j^L$  is the market share of firm  $j$  in the labor market; and the WIPI is the worker internal power index, defined as:

$$\text{WIPI} \equiv \sum_j \frac{\int_{i \in I} \gamma_{ij} \lambda_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j^L,$$

where  $\lambda_i$  is the fraction of employment supplied by shareholder  $i$ .

*Proof.* In Appendix. ■ □

We can also rewrite the WIPI in terms of the cosine similarities and variances:

**Proposition 5.** *The WIPI can be expressed in terms of the variance of labor supply shares, the variances of the ownership shares of the various firms, and the ratios of cosine similarities between labor supply and control shares and between ownership and control shares:*

$$\text{WIPI} = \sum_j \frac{\cos(\alpha_{\gamma_j, \lambda}) \sqrt{\sigma_\lambda^2 + 1}}{\cos(\alpha_{\gamma_j, \beta_j}) \sqrt{\sigma_{\beta_j}^2 + 1}} s_j^L,$$

where  $\alpha_{\gamma_j, \lambda}$  is the angle between the labor supply share vector  $\lambda$  and the control share vector of firm  $j$ ,

$\gamma_j$ ,  $\alpha_{\gamma_j, \beta_j}$  is the angle between the ownership share vector and the control share vector of firm  $j$ , and  $\sigma_\mu^2$  and  $\sigma_{\beta_j}^2$  are the variances of the labor supply share and ownership share vectors, respectively.

*Proof.* In Appendix. ■ □

As in the previous model, internalization of labor supply effects are likely to be small under pure shareholder capitalism with an infinite variance of wealth.<sup>13</sup> However, in mixed economies that combine elements of shareholder capitalism with some worker representation, many institutional arrangements have emerged to provide workers direct control in firms beyond the control that their shareholdings would provide. Internalization of labor supply is likely to have significant effects when such institutional arrangements are present, with the effect of increasing wages and bringing them closer to the marginal product of labor. Examples of institutional arrangements to increase the correlation between labor supply and control of firms include co-determination (some level of board representation for employees in private companies is mandated by law in Austria, Croatia, Denmark, Finland, France, Germany, Hungary, Luxembourg, the Netherlands, Norway, Slovakia, Slovenia and Sweden; see [Fulton, 2011](#)), Employee Stock Ownership Programs (ESOP) in the United States, and worker cooperatives. Note that these institutional arrangements, unlike antitrust policy, do not attempt to reduce market power in the labor market, but simply to mitigate its effect on wages by giving some control to those affected by the market power. Institutions such as unions or policies such as the minimum wage may have similar effects in practice, but the mechanisms through which they affect wages—collective bargaining and direct regulation, respectively—are different from worker representation in the objective function of the firm. The following example illustrates how co-determination, under some assumptions, can restore efficiency by having the firm's objective function internalize the effects of its market power over its own employees.

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<sup>13</sup>Including human capital in the model is unlikely to change this conclusion, given that income is approximately log-normally distributed and therefore has a finite variance.

## 5.1 Application: optimal level of worker representation in corporate decisions

In many countries, the level of legally mandated representation of employees in corporate boards is a function of firm size, with larger firms having as much as half of the seats assigned to labor. We show here that this idea follows naturally from the model of oligopsony with shareholder workers, with one modification: we consider the case in which the employees have an exogenously determined share  $\gamma^{L,j}$  of control at firm  $j$ .

For simplicity, we consider the case in which firms are separately owned, and the employees and shareholders are different groups, each of measure one. Assume that all the members within each group are identical. In the absence of co-determination, the shareholders have full control of the firms. Each representative shareholder has one share in the firm. Assume that, apart from the ownership or not of the firms, agents are otherwise identical. The equilibrium markdown of wages below the marginal product of labor at firm  $j$  without co-determination is

$$\frac{F'_j - \frac{w}{p}}{\frac{w}{p}} = \frac{s_j^L}{\eta^L}.$$

Note that larger firms have larger markdowns, even though in this model real wages are the same across all firms. The reason is that more productive firms endogenously have higher market shares in the model. Consider now the case in which the employees have a control share of  $\gamma_{L,j}$ . The objective of firm  $j$ 's managerial candidates is now

$$\max_{L_j^d} (1 - \gamma_{L,j})v(w, m_{i,0} + \pi_j) + \gamma_{L,j}v(w, m_{i,0} + wT) + k,$$

The first-order condition of the firm is now

$$(1 - \gamma_{L,j}) \left( pF'_j - w - w'L_j^d \right) + \gamma_{L,j} (w'L_i).$$

To make the markup equal to zero requires

$$(1 - \gamma_{L,j}) \frac{L_j^d}{L} = \gamma_{L,j} \frac{L_i}{L}.$$

Since the workers have measure one and are all identical  $\frac{L_i}{L} = 1$ , this is the same as

$$s_j^L = \frac{\gamma_{L,j}}{1 - \gamma_{L,j}}.$$

Solving for  $\gamma_{L,j}$  in terms of the employment share of firm  $j$ :

$$\gamma_{L,j} = \frac{s_j^L}{1 + s_j^L}.$$

For example, if the firm is a monopsonist, then efficiency is restored when employees are assigned half of the board seats. In the case of duopsonist with a 50% market share, efficiency is restored when employees are assigned a third of the board seats.

Consider now a more general case, in which labor supply is still homogeneous, but shareholders hold arbitrary portfolios and are now also workers of the firms. To keep the analysis simple, we assume that every agent supplies the same amount of labor to the firms, and we assume that workers are assigned randomly to firms, so that the distribution of shareholdings among the workers of every firm is the same. The set of shareholders that are also workers in firm  $j$  is denoted  $I_j$ . In this case, shareholder  $i$  has a control share of  $(1 - \gamma_{L,j})\gamma_{ij}$  from its holdings in firm  $j$ , and an additional control share of  $\frac{\gamma_{L,j}}{s_j^L}$  if it is a worker of firm  $j$ . The objective function of the firm is now:

$$\max_{L_j^d} \int_{i \in I} (1 - \gamma_{L,j})\gamma_{ij} v_i \left( w(L_j^d; L_{-j}^d), m_i(L_j^d; L_{-j}^d) \right) di + \int_{i \in I_j} \frac{\gamma_{L,j}}{s_j^L} v_i \left( w(L_j^d; L_{-j}^d), m_i(L_j^d; L_{-j}^d) \right) di.$$

The assumption that shareholders are assigned randomly to firms implies that  $\int_{i \in I_j} v_i di = s_j^L \int_{i \in I} v_i di$ , because the distribution of  $v_i$  is the same for the set of shareholders  $I_j$  as for the

set of all shareholders, and therefore  $\int_{i \in I_j} v_i di$  is just the average of  $v_i$  times the measure of set  $I_j$ . Thus, the objective function of firm  $j$  simplifies to:

$$\max_{L_j^d} \int_{i \in I} ((1 - \gamma_{L,j})\gamma_{ij} + \gamma_{L,j}) v_i \left( w(L_j^d; L_{-j}^d), m_i(L_j^d; L_{-j}^d) \right) di.$$

In this case, we can show that at each firm there is a level of co-determination that would return to an efficient outcome, that is, wage equal to the value of the marginal product of labor:

**Proposition 6.** *Suppose that labor supply is homogeneous, workers are randomly assigned to firms, and the distribution of control among the shareholders is proportional to their ownership shares. There exists a level of worker representation at each firm  $\gamma_{L,j}^* \in [0, 1]$  such that the firm sets wage equal to the value of the marginal product of labor.*

*Proof.* In Appendix. ■

□

This shows that some level of board representation for workers can help address the market power by firms in the labor market. In the presence of common ownership the exact level that would achieve this depends on the ownership structure of the firm in a non-trivial way, and not just on the firm's size. Solving for the optimal level of co-determination would be more difficult in the case of heterogenous workers or non-random assignment of workers to firms (this would be the case, for example, if ownership of a particular firm is correlated with being assigned to that firm). However, it seems that even in these more general cases the idea that *some* level of employee representation can help mitigate the effects of market power in labor markets, especially at large firms, would still hold.

## 6 Conclusion

This paper developed a model of firm behavior in the context of portfolio diversification and market power by the firms. Various potential managers compete for shareholder votes. When

firms have overlapping owners, they take into account the impact of their strategic plans not only on their own profits, but also on the profits of rivals that are on their shareholders' portfolios. The model shows a straightforward mechanism that implies that direct communication between shareholders and managers is not necessary for overlapping ownership to have an effect on firm behavior, as long as shareholders have the power of the vote and their portfolios are public information. We used the voting model to derive an endogenous objective function of the firm, and expressions for shareholder control over the oligopolistic firms. We then used this framework to prove, under alternative assumptions, versions of a "folk theorem:" the idea that if all shareholders hold perfectly diversified portfolios then all firms act as a single firm.

Applying the framework to the setting of oligopoly with shareholder-consumers, we derived an expression for the average industry markup in terms of the difference between a modified Herfindahl-Hirschman Index and an index that quantifies the alignment of consumption and control of the firms. In the context of labor-market oligopsony, we obtain a similar expression for the average markdown of wages below the value of the marginal product of labor. We argued that, given that the variance of stock market wealth is much higher than the variance of consumption or labor supply, these effects are likely to be small under pure shareholder capitalism, but other institutional arrangements and corporate governance institutions can provide additional worker control. Leading examples include worker and consumer cooperatives, employee representation in corporate boards, and employee stock ownership programs. These institutions can in principle help mitigate the negative effects of market power, even though they do not reduce that power. Antitrust policy, on the other hand, by reducing common ownership can reduce the markdown of wages relative to the marginal product of labor through an increase in competition for consumers and workers by firms.<sup>14</sup> To the extent that antitrust policy cannot completely eliminate market power—especially in labor markets—these kinds of institutional arrangements could be a complementary way to address the problem of market

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<sup>14</sup>See [Azar \(2011, 2012\)](#), [Azar, Schmalz, and Tecu \(2015\)](#), [Posner and Weyl \(2015\)](#), [Elhauge \(2016\)](#), [Baker \(2016\)](#), [Azar, Raina, and Schmalz \(2016\)](#), and [Posner, Scott Morton, and Weyl \(2016\)](#) for further analysis and policy proposals on antitrust policy in relation to portfolio diversification and common ownership by financial institutions.

power.

The theory of the firm developed in this paper differs from previous theories in that firm boundaries can become blurred through common ownership links (as well as its focus on market power). Previous theories, on the other hand, were essentially theories of mergers ([Coase, 1937](#); [Williamson, 1975](#); [Grossman and Hart, 1986](#); [Hart and Moore, 1990](#)). Portfolio diversification can lead to the integration of firm ownership without economic or strategic objectives other than portfolio risk reduction. When all agents hold market portfolios, the shareholders want the firms to act in unison to maximize their objectives, and firms essentially become branches of a larger corporate structure, even if they are formally separate entities. Thus, theorists of firm boundaries may need to consider not just why different ownership structures may arise, but also why organization within a common ownership “superorganism” would require separate firms with highly correlated ownership rather than just one large superfirm that is formally a unit. Possible answers include efficiencies from the elimination of redundancies, and that attempting to formally merge all publicly traded firms would push against the antitrust laws in an obvious way, while portfolio diversification does so in a way that could more easily “go under the radar,” or even when it is considered, may be harder to tackle by the antitrust authorities.

The theory of firm behavior developed in this paper is particularly well suited for macroeconomic modeling and general equilibrium analysis more broadly. The reason is that, in contrast with profit maximization as an assumption for firm behavior, firm strategies that are the outcome of a voting process are independent of the choice of numeraire. In that direction, [Azar and Vives \(2016\)](#) develop a simple macroeconomic model with large firms and shareholder voting to explore the effect of market concentration on aggregate outcomes. Studying further the macroeconomic implications of changes in market power could be a promising area for future research.

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## A Mathematical Appendix

### PROOF OF PROPOSITION 1:

The conditional sincerity assumption ensures that the maximization problem of a managerial candidate can be expressed taking as given the strategies of the other firms. The assumptions of concavity, uniformly distributed bias, and vote-share objective ensure that the objective function of each managerial candidate is strictly concave conditional on rival firm strategies. Together with the assumption of compact strategy sets, this ensures that each candidate's maximization problem has a unique maximum. Because the maximization problems of both candidates at each firm are symmetric, they both choose the same strategy conditional on the

strategies of the other firms. This leads to a well-defined best-response function for each firm, as a function of the strategies of rival firms. The assumptions of compact-convex strategy sets and continuity ensure that the best-response functions are upper-hemicontinuous. We can thus apply Kakutani's fixed point theorem to ensure existence of an equilibrium. ■

PROOF OF THEOREM 1:

We start by noting that, when all shareholders are completely diversified,  $\beta_{in}$  is the same across firms for a given shareholder. That is,  $\beta_{in} = \beta_i$  for all  $n$  and all  $i$ . Thus, the outcome of the voting equilibrium is characterized by the solution to

$$\max_{\mathbf{p}^n} \sum_{i \in I} \psi_{in} \beta_i u_i \left( \beta_i \sum_{s=1}^N \pi_s(\mathbf{p}_s, \mathbf{p}_{-s}) \right) \text{ for } n = 1, \dots, N.$$

We can rewrite this as

$$\max_{\mathbf{p}^n} f_n \left( \sum_{s=1}^N \pi_s(\mathbf{p}_s, \mathbf{p}_{-s}) \right) \text{ for } n = 1, \dots, N,$$

where

$$f_n(z) = \sum_{i \in I} \psi_{in} \beta_i u_i(\beta_i z).$$

Since  $f_n(z)$  is monotonically increasing, the solution to is equivalent to

$$\max_{\mathbf{p}^n} \sum_{s=1}^N \pi_s(\mathbf{p}_s, \mathbf{p}_{-s}) \text{ for } n = 1, \dots, N.$$

Because the objective function is the same for all firms, we can rewrite this as

$$\max_{\{\mathbf{p}^n\}_{n=1}^N} \sum_{s=1}^N \pi_s(\mathbf{p}_s, \mathbf{p}_{-s}).$$

■

PROOF OF THEOREM 2:

Because of complete diversification, a shareholder  $i$  holds the same number of shares  $\beta_i$  in

each firm. The equilibrium thus corresponds to the solution of

$$\max_{\mathbf{p}^n} \sum_{i \in I} \psi_{in} \beta_i U_i(\mathbf{p}_n, \mathbf{p}_{-n}) \text{ for } n = 1, \dots, N.$$

Assumption 7 requires  $\psi_{in}$  to be the same for every firm, and thus the objective function is the same for all  $n$ . The problem can thus be rewritten as

$$\max_{\{\mathbf{p}^n\}_{n=1}^N} \sum_{i \in I} \psi_i \beta_i U_i(\mathbf{p}_n, \mathbf{p}_{-n}).$$

The solution to this maximization problem corresponds to that of a monopolist with a weighted average of the utilities of the shareholders as its objective function. The weight of shareholder  $i$  is equal to  $\psi_i \beta_i$ . ■

#### PROOF OF PROPOSITION 2:

We take as the starting point for the proof the first-order condition after applying Roy's identity:

$$\int_{i \in I} \gamma_{ij} \left\{ p'(Q) \left( \sum_k \beta_{ik} q_k - x_i(p(Q)) \right) + \beta_{ij} \left( p(Q) - c'_j(q_j) \right) \right\} di = 0.$$

Bringing the integral inside the sum yields:

$$p'(Q) \left( \sum_k q_k \int_{i \in I} \gamma_{ij} \beta_{ik} di - \int_{i \in I} \gamma_{ij} x_i(p(Q)) di \right) + \int_{i \in I} \gamma_{ij} \beta_{ij} di \left( p(Q) - c'_j(q_j) \right) = 0.$$

Dividing by  $\int_{i \in I} \gamma_{ij} \beta_{ij} di$  and rearranging terms, we can rewrite this as:

$$p(Q) - c'_j(q_j) = -p'(Q) \left\{ \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} q_k - \frac{\int_{i \in I} \gamma_{ij} x_i(p(Q)) di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \right\}.$$

Dividing both sides by  $p$  and multiplying and dividing the right-hand side by  $Q$  yields an expression for the markup of firm  $j$  in terms of shares and the elasticity of demand (to make

the notation less cumbersome, here we leave implicit that price and cost are functions of the quantities, and that shareholder  $i$ 's demand is a function of price):

$$\frac{p - c'_j}{p} = -\frac{p'Q}{p} \left\{ \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \frac{q_k}{Q} - \frac{\int_{i \in I} \gamma_{ij} \frac{x_i}{Q} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \right\}.$$

Using the fact that  $s_k = \frac{q_k}{Q}$ ,  $\mu_i = \frac{x_i}{Q}$ , and  $\frac{1}{\eta} = -\frac{p'Q}{p}$ , we can rewrite this expression as

$$\frac{p - c'_j}{p} = \frac{1}{\eta} \left\{ \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_k - \frac{\int_{i \in I} \gamma_{ij} \mu_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \right\}.$$

Taking a market-share weighted average on both sides yields

$$\sum_j s_j \frac{p - c'_j}{p} = \frac{1}{\eta} \left( \sum_j \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j s_k - \sum_j \frac{\int_{i \in I} \gamma_{ij} \mu_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j \right),$$

which completes the proof. ■

### PROOF OF PROPOSITION 3:

Take the formula for the CIPI and use the inner product formula

$$\int a_i b_i di = \cos(\alpha_{a,b}) \sqrt{\int a_i^2 di} \sqrt{\int b_i^2 di},$$

to rewrite it as

$$CIPI = \sum_j \frac{\cos(\alpha_{\gamma_j, \mu}) \sqrt{\int \gamma_{ij}^2 di} \sqrt{\int \mu_i^2 di}}{\cos(\alpha_{\gamma_j, \mu}) \sqrt{\int \gamma_{ij}^2 di} \sqrt{\int \beta_{ij}^2 di}} s_j.$$

The  $\sqrt{\int \gamma_{ij}^2 di}$  factors cancel out. Adding and subtracting one inside the square roots in the other factors yields:

$$CIPI = \sum_j \frac{\cos(\alpha_{\gamma_j, \mu}) \sqrt{\int \mu_i^2 di - 1 + 1}}{\cos(\alpha_{\gamma_j, \mu}) \sqrt{\int \beta_{ij}^2 di - 1 + 1}} s_j.$$

Noting that  $\sigma_\mu^2 = \int \mu_i^2 di - 1$  and  $\sigma_{\beta_j}^2 = \int \beta_{ij}^2 di - 1$  completes the proof. ■



PROOF OF PROPOSITION 4:

We take as the starting point for the proof the first-order condition after applying Roy's identity:

$$\int_{i \in I} \gamma_{ij} \left\{ w'(L) \left( L_i(w(L)) - \sum_k \beta_{ik} L_k^d \right) + \beta_{ij} \left( pF'_j(L_j^d) - w(L) \right) \right\} di = 0.$$

Bringing the integral inside the sum yields:

$$w'(L) \left( \int_{i \in I} \gamma_{ij} L_i(w(L)) di - \sum_k \int_{i \in I} \gamma_{ij} \beta_{ik} di L_k^d \right) + \int_{i \in I} \gamma_{ij} \beta_{ij} di \left( pF'_j(L_j^d) - w(L) \right) = 0.$$

Dividing by  $\int_{i \in I} \gamma_{ij} \beta_{ij} di$  and rearranging terms, we can rewrite this as:

$$pF'_j(L_j^d) - w(L) = w'(L) \left( \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} L_k^d - \frac{\int_{i \in I} \gamma_{ij} L_i(w(L)) di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \right).$$

Dividing both sides  $w$  and multiplying and dividing the right-hand-side by  $L$  yields an expression for the markdown of firm  $j$  in terms of the employment shares and the elasticity of labor supply (to make the notation less cumbersome, here we leave implicit that the wage and the production function are functions of the labor quantities, and that the labor supply of shareholder  $i$  is a function of the wage):

$$\frac{pF'_j - w}{w} = \frac{w'L}{w} \left( \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \frac{L_k^d}{L} - \frac{\int_{i \in I} \gamma_{ij} \frac{L_i}{L} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \right).$$

Using the fact that  $s_k^L = \frac{L_k^d}{L}$ ,  $\lambda_i = \frac{L_i}{L}$ , and  $\frac{1}{\eta^L} = \frac{w'L}{w}$ , we can rewrite this expression as

$$\frac{pF'_j - w}{w} = \frac{1}{\eta^L} \left( \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_k^L - \frac{\int_{i \in I} \gamma_{ij} \lambda_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} \right).$$

Taking an employment-share weighted average on both sides yields

$$\sum_j s_j^L \frac{pF_j' - w}{w} = \frac{1}{\eta^L} \left( \sum_j \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j^L s_k^L - \sum_j \frac{\int_{i \in I} \gamma_{ij} \lambda_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j^L \right).$$

To complete the proof, we rewrite the markdown on the left-hand side in terms of the real wage, by dividing by the price in the numerator and the denominator:

$$\sum_j s_j^L \frac{F_j' - \frac{w}{p}}{\frac{w}{p}} = \frac{1}{\eta^L} \left( \sum_j \sum_k \frac{\int_{i \in I} \gamma_{ij} \beta_{ik} di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j^L s_k^L - \sum_j \frac{\int_{i \in I} \gamma_{ij} \lambda_i di}{\int_{i \in I} \gamma_{ij} \beta_{ij} di} s_j^L \right).$$

■

#### PROOF OF PROPOSITION 5:

The proof is based on the formula for the inner product in terms of the cosine similarity and the norms, and is similar to the proof of the analogous formula for the CIPI. ■

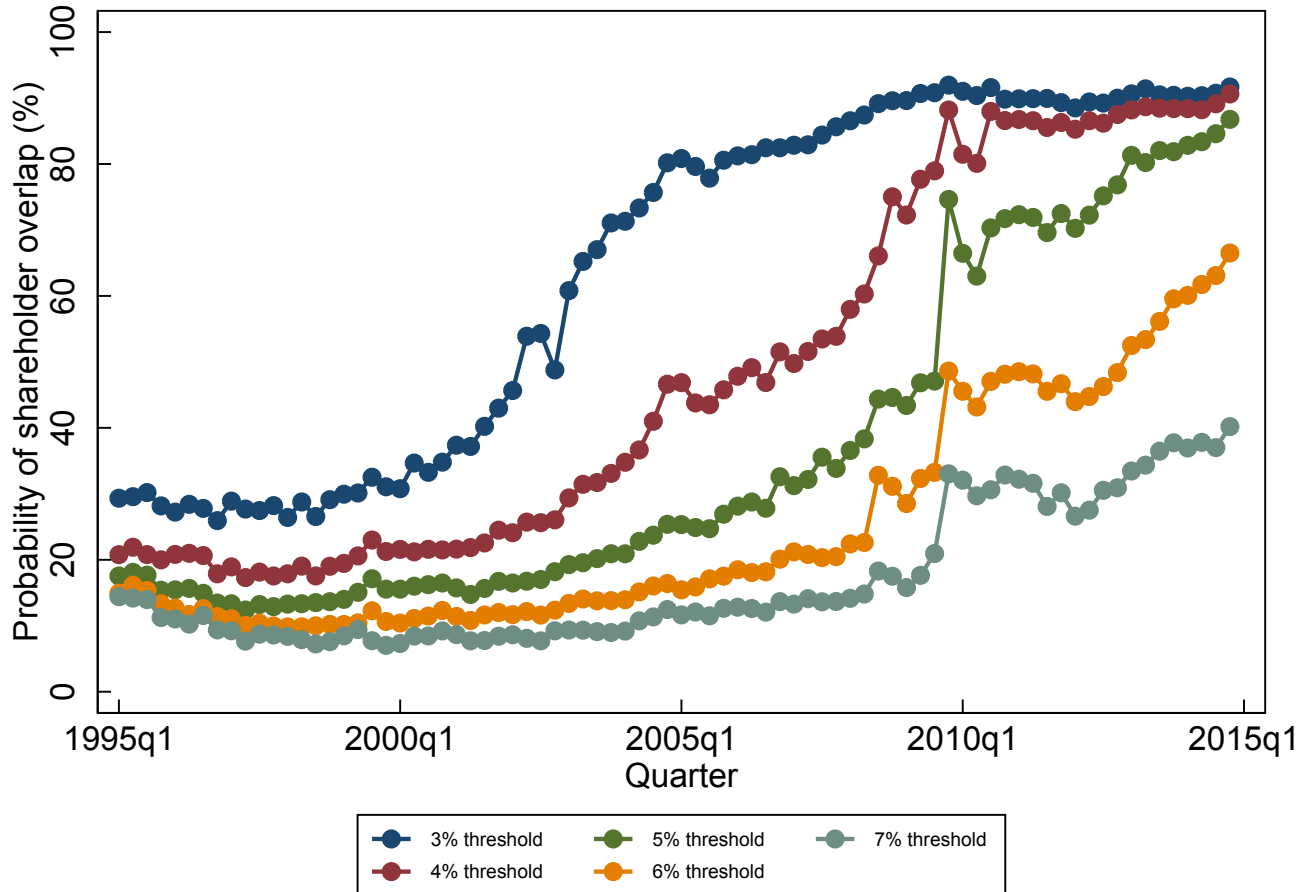
#### PROOF OF PROPOSITION 6:

The problem is equivalent to taking the formula for the markup derived in proposition 4, but with control shares  $\gamma_{L,j} + (1 - \gamma_{L,j})\gamma_{i,j}$ . From the first-order condition of firm  $j$ , the wage is equal to the value of the marginal product if and only if

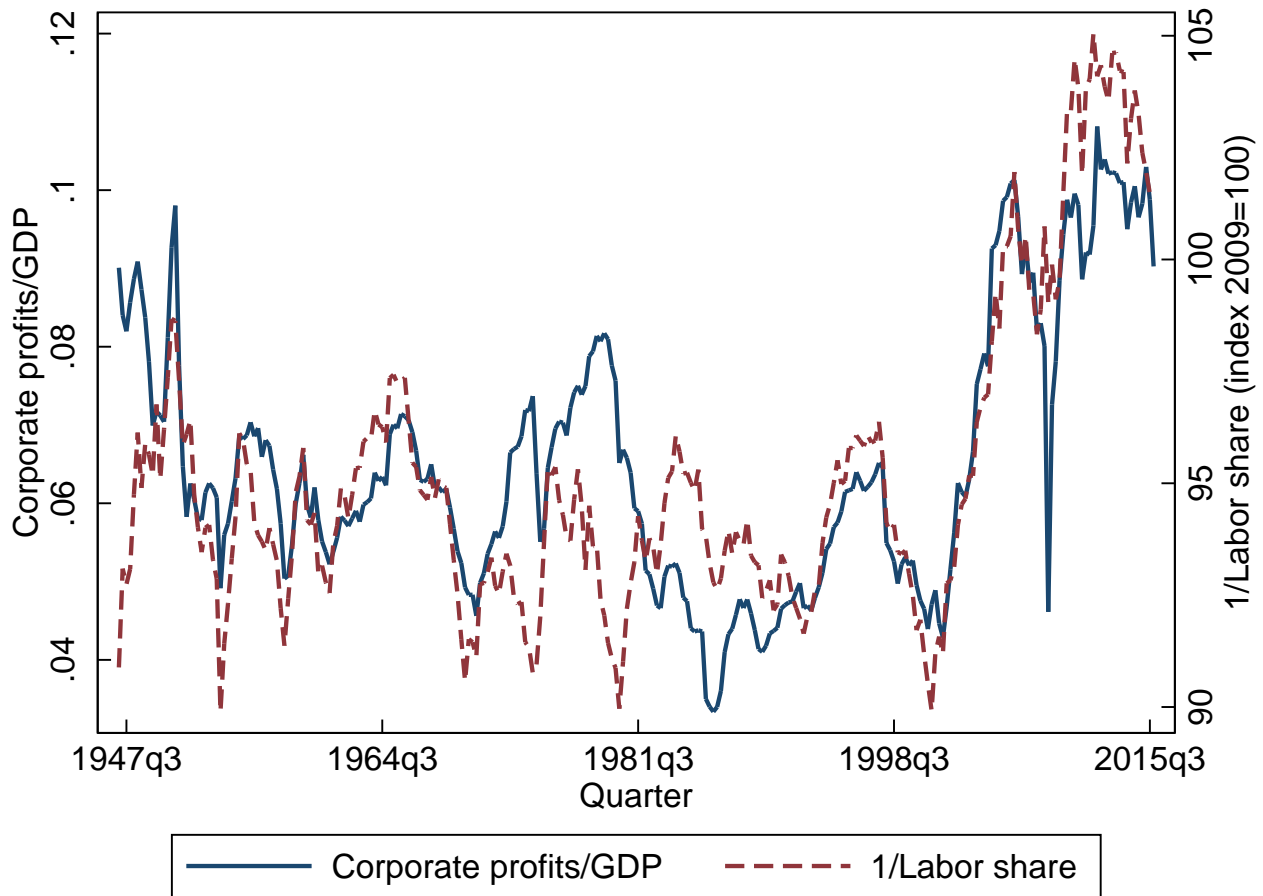
$$(1 - \gamma_{L,j}) \left( s_j^L \int_{i \in I} \beta_{ij}^2 di + \sum_{k \neq j} s_k^L \int_{i \in I} \beta_{ij} \beta_{ik} di - 1 \right) - \gamma_{L,j} = 0$$

Since  $\int_{i \in I} \beta_{ij}^2 di \geq 1$ , this expression is non-negative if  $\gamma_{L,j} = 0$ . The expression is negative if  $\gamma_{L,j} = 1$ . Since this is a continuous function of  $\gamma_{L,j}$ , the intermediate-value theorem implies that there is a  $\gamma_{L,j}$  such that it is equal to zero, and therefore for this  $\gamma_{L,j}$  the wage equals the marginal product at firm  $j$ . ■

## Figures



**Figure 1. Probability of shareholder overlap for S&P 1500 firms in the same industry at various ownership thresholds.** This figure shows the probability that two firms in the same 3-digit SIC industry, selected at random from the S&P 1500 index component firms (and that are also in the Thomson dataset), have a shareholder in common (with at least  $x\%$  ownership in both firms, where  $x = 3, 4, 5, 6,$  or  $7$  percent) over the period 1995Q1 to 2014Q4. The probability is sometimes also called the density of the interlocking shareholder network:  $Density = \frac{\sum_{i=1}^n \sum_{j < i} y_{ij}}{n(n-1)/2}$ , where  $n$  is the number of nodes in the network and  $y_{ij}$  is equal to 1 if firm  $i$  and firm  $j$  are connected, and zero otherwise (by convention, a firm is not considered to be connected to itself, and thus the adjacency matrix has zeros in its diagonal). The probability in a given period and for a given threshold is calculated by first calculating the network density for each industry, and then taking a weighted average of the densities with the number of firms in each industry as weights. Source: author's calculations using data from Thomson 13F Institutional Ownership and SEC filings for Barclays and BlackRock in periods where some of their filings are missing from the Thomson data.



**Figure 2. Corporate profits as a share of GDP and inverse of the labor share: 1947Q1–2015Q3.** This figure shows corporate profits after tax as a share of GDP and the inverse of the labor share for the nonfinancial corporate sector over the period 1947Q1–2015Q3. The inverse of the labor share is often used as a proxy for the aggregate level of markups (Rotemberg and Woodford, 1999; Nekarda and Ramey, 2010). Source: author’s calculations using data from FRED.