THE IMPACT OF THE VALUE ADDED TAX
ON A DIFFERENTIATED DUOPOLY

Jordi Gual
THE IMPACT OF THE VALUE ADDED TAX ON A DIFFERENTIATED DUOPOLY

Jordi Gual

Abstract

This paper studies how changes in value added tax rates affect the interactions between producers of differentiated products. The paper focuses on duopolists competing in international markets.

The results show that, by and large, tax rate differentials lead to price discrimination against the low-tax countries. It is shown that market integration usually results in global welfare improvements. When transportation costs are significant, increasing the tax rate tends to increase the market share of domestic producers. However, this improvement in the competitive position of domestic producers cannot result, in general, in larger profits.

Note: This paper draws on the first chapter of my Ph.D. Dissertation (U.C. Berkeley, 1987). I am grateful to Professors Pranab Bardhan, Thomas Rothenberg and Jeffrey Perloff for their support. Comments from Carmen Matutes and participants at the Fourteenth Annual Conference of the European Association for Research in Industrial Economics, August 30-September 1, 1987 (Madrid) have proved very useful. However, the usual disclaimer applies. Financial support from the Institute of International Studies (U.C. Berkeley) and from the Commission of the European Communities is also gratefully acknowledged.


1 Professor of Economics, IESE
THE IMPACT OF THE VALUE ADDED TAX ON A DIFFERENTIATED DUOPOLY

1. Introduction

The purpose of this paper is to study how changes in value added tax (VAT) rates affect the interactions between producers of differentiated products. The paper focuses on duopolists competing in international markets. This is an issue of special interest in the European Community since the goal of tax rate harmonization has not been achieved so far (see, for example, Table 1 for VAT rates in some automobile markets), and some observers have argued that these differentials can hinder intra-Community trade.

Table 1
Value Added Tax Rates for Selected EEC Countries

<table>
<thead>
<tr>
<th></th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium *</td>
<td>25</td>
</tr>
<tr>
<td>Denmark*</td>
<td>22</td>
</tr>
<tr>
<td>France</td>
<td>33.33</td>
</tr>
<tr>
<td>German *</td>
<td>14</td>
</tr>
<tr>
<td>Italy *</td>
<td>20</td>
</tr>
<tr>
<td>Netherlands*</td>
<td>19</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>12</td>
</tr>
</tbody>
</table>

1Belgium: the tax is 33% if the car horsepower is greater than 116kw, or the engine size exceeds 3000cc.
Denmark: there is also a registration tax on the VAT-included price. The rate is 105% for the first 19750 Krones and 180% for the rest.
Italy: the tax is 38% if the engine size exceeds 2000cc (2500 for diesel engines).
Netherlands: there is also an additional tax on the VAT-excluded price. The rate is 16% for 100/119 of the first 10000 Florins and 24% for 100/119 of the rest. It is 16% of the price if it is not greater than 10000 Florins.
United Kingdom: there is also a Special Car Tax of 10% computed on 5/6 of the retail price.

The results of this paper show that, by and large, tax rate differentials lead to price discrimination against the low-tax countries. Thus, tax rates divergences have to be taken into account when studying the sources of net price differentials arising in economically integrated areas such as the European Community. For example, Mertens and Ginsburgh (1985) have neglected this influence. Net price differentials have been as large as 50% for certain products and countries. Related work by the author (Gual, 1987) specifically incorporates tax rate differentials as an explanation of net price disparities in the Community. There, it is found that the tax variable contributes significantly to the existence of price gaps. Specifically, a 1% increase in the tax differential leads to a positive but proportionally smaller decrease in the net price differential of around 10-15 ECUs.

This paper also explores the welfare consequences of a more integrated market. Under most circumstances, integration – as defined below – leads to a welfare improvement for both countries.

Finally, this paper shows that, when transportation costs are significant, increasing the tax rate tends to increase the market share of domestic producers. However, this improvement in the competitive position of domestic producers cannot result, in general, in larger profits.

2. The Model

We consider a very simple model where two differentiated products (x,y) are manufactured by two duopolists. The following direct demand system can be easily derived from a utility maximization problem of a representative consumer\(^1\):

\[
\begin{align*}
\frac{\partial x}{\partial P} & = x(P,Q) \\
\frac{\partial y}{\partial Q} & = y(P,Q)
\end{align*}
\]

where \(P\) is the market price of \(x\) and \(Q\) is the market price of \(y\). This system will have the following properties: it is continuous in \(R^2\); twice-continuously differentiable in the interior of the region in price space where \(x>0, y>0\); decreasing in its own arguments, \(\partial_x x < 0, \partial_x y < 0\); with symmetric and positive cross-price effects \(\partial_x x = \partial_y y > 0\) (substitute goods); and with dominance of own-price effects.

We take the number of firms as given and assume that entry is prevented either by institutional constraints or by the presence of economies of scale due to the existence of large fixed costs \((F_1,F_2)\). Specifically, \(F_i\) will be assumed to be at a level that allows both firms (and only them) to make positive profits.

For the moment we will assume that firms have constant and equal marginal production costs. As we will see, relaxing this assumption has important consequences.

Firms will be assumed to compete in prices in a Bertrand fashion. Each firm maximizes its own profit function by choice of price, taking the price choice of its rival as given. A Nash equilibrium in prices will be a price pair \((p,q)\) such that no firm can do better by deviating. If

---


\(^2\) \(\partial_i x\) will denote the partial derivative of the function \(x\) with respect to the \(i^{th}\) argument.
marginal costs are constant, a Bertrand equilibrium in pure strategies is known to exist in general, provided the profit functions are quasi-concave.

Finally, we will also assume that both firms can make positive profits even when their rival’s price equals marginal cost. This will insure that the equilibrium is interior.

We will denote the value added tax rate by $t$. The implementation of VAT in the EEC suggests that the net revenue functions of the duopolists can be specified as:

$$NR_1(p,q,(1+t)) = px(P,Q)$$
$$NR_2(p,q,(1+t)) = qy(P,Q)$$

where $(p,q)$ are net prices and $P = p(1+t)$, $Q = q(1+t)$. This implies that firms quote net prices. The actual transaction price includes the tax surcharge but all extra revenue is tax liability $Ptx(P,Q)$. In this context the strategic variables are net prices, but of course the players recognize the negative impact of the tax rate on demand. We will label this specification the **surcharge model**.

Alternatively, the tax could be collected as a percentage of gross revenue, with firms quoting final market prices. The net revenue functions would be:

$$NR_1(P,Q,t) = (1-t) P x(P,Q)$$
$$NR_2(P,Q,t) = (1-t) Q y(P,Q)$$

This would seem a more correct implementation of the VAT since the tax liability $Ptx(P,Q)$, is a percentage of gross revenues (for simplicity, equal to gross value added in this problem). Market prices are the strategic variables of the firms in this **"ad valorem" model**.

The two possible VAT implementations have similar impacts on market equilibrium. The main features of the **"ad valorem" model** are considered in the Appendix.

### 3. The Effect of Changing VAT Rates on Equilibrium Prices

#### 3.1. Net Prices

We will concentrate on the analysis of the impact of higher VAT rates in the surcharge model, which seems an appropriate representation of the VAT in its current implementation. The following proposition is the central result of this paper:

**Proposition 1.** An increase in the VAT rate will cause a decrease in net equilibrium prices if the demand functions are not too convex in their own price. When the products are strategic complements we also require non-positive cross-partial derivatives of the demand functions.

---

3 Internal EEC regulations allow us to abstract from the impact of VAT at intermediate stages of production since firms are entitled to a full refund of all taxes paid on inputs.

4 This is the usual way to model the impact of **"ad valorem"** tariffs in this type of model (see Eaton and Grossman (1983) and Krishna (1985); the papers of Itoh and Ono (1982), (1984) are exceptions to this procedure).
To show this proposition, consider the following profit functions:

\[ \pi_1(p, q, (1 + t)) = (p - k)x(P, Q) - F_1 \]
\[ \pi_2(p, q, (1 + t)) = (q - k)y(P, Q) - F_2 \]

Define the own-price elasticities for both goods as follows:

\[ \phi_1(P, Q) = \left[ -P \partial_{x}x \right] / x; \phi_2(P, Q) = \left[ -Q \partial_{y}y \right] / y; \]

The first-order conditions for this problem can then be written as:

\[ p \left[ 1 - \left( 1/\phi_1 \right) \right] = k \]
\[ q \left[ 1 - \left( 1/\phi_2 \right) \right] = k \]  \hspace{1cm} (1)

To ensure the uniqueness and stability of this equilibrium we make the following additional assumption:

\[ \partial_{11} \pi_1 \partial_{22} \pi_2 > \partial_{21} \pi_2 \partial_{12} \pi_1 \]

for all prices in the region of price space where \( x > 0 \) and \( y > 0 \).

This assumption will also ensure that the second-order conditions are satisfied and implies that the reaction functions are well-behaved and have slope of less than one in absolute value.

We are interested in the sign of \( dp/d(l + t) \).\(^5\) Totally differentiating the first-order conditions and solving yields the following:

\[ dp/d(1 + t) = \left[ \partial_{12} \pi_1 \partial_{13} \pi_1 - \partial_{22} \pi_2 \partial_{23} \pi_1 \right] / \Omega \]  \hspace{1cm} (2)

where \( \Omega = \partial_{11} \pi_1 \partial_{22} \pi_2 - \partial_{22} \pi_2 \partial_{12} \pi_1 \), and \( \partial_{jk} \pi_1 \) denotes the second partial derivative of the \( i \)th profit function with respect to the \( j \)th and \( k \)th argument.

By symmetry \( \partial_{13} \pi_1 = \partial_{23} \pi_2 \) and \( \partial_{21} \pi_2 = \partial_{12} \pi_1 \). Then we can write \( (2) \) as:

\[ dp/d(1+t) = \left[ \partial_{21} \pi_2 - \partial_{22} \pi_2 \right] \partial_{13} \pi_1 / \Omega \]  \hspace{1cm} (3)

The fulfillment of the second-order conditions implies that \( \Omega \) is larger than zero. Similarly, the stability conditions require: \( \partial \partial_{j} \pi_1 + \partial \partial_{k} \pi_1 < 0 \). Then \( \left[ \partial_{21} \pi_2 - \partial_{22} \pi_2 \right] > 0 \) and the sign of \( dp/d(1 + t) \) will be that of \( \partial_{13} \pi_1 \).

This sign can be easily established by noting that:

\[ \partial_{13} \pi_1 = \left[ (1 + t) \partial_{1} x \left[ \partial_{(1+t)(1)} \phi(P, Q) \right] \right] / \phi_1 \]  \hspace{1cm} (4)

We have then:

\[ \text{sign} \ [dp/d(l+t)] = - \text{sign} \ \partial_{(1+t)} \phi_1 = - \text{sign} \ \left[ p \partial_{(1)} \phi_1 + q \partial_{2} \phi_1 \right] \]

\(^5\) By symmetry \( dp/d(l+t) = dq/d(1+t) \), and we can concentrate on the price of \( x \).
A sufficient condition for $\partial_1 \phi_1 > 0$ is that the demand function is not too convex with respect to its own price. As for $\partial_1 \phi_1$, when marginal costs are constant it will be positive if the two goods are strategic substitutes. That is, if an aggressive move by one player triggers an accommodative response of its rival. It is negative if the products are strategic complements. However, a sufficient condition that guarantees a positive sign for $\partial l \phi_1$ is that $\partial_1 x \leq 0$. That is, if an increase in the price of the rival firm does not decrease the own-price slope of demand.

The importance of Proposition 1 is that the conditions imposed on the demand system are not unduly restrictive. Thus, we would expect this kind of outcome on most occasions as long as the net revenue functions correctly reflect the process of tax collection and the pricing behavior of businesses.

The source of this downward movement in net prices is that higher taxes have a negative impact on marginal profitability, thus leading to an inward shift in the reaction functions. This downward movement will be compounded if the goods are strategic complements since price decreases of the rival firm trigger further own-price reductions.

What is essentially happening is that higher taxes, under the conditions stated in the result, tend to make demand for the good more elastic. With constant marginal costs, firms will tend to lower prices to restore the equilibrium.

The intuition behind the negative impact on marginal profitability might be clarified by studying the linear case in the example that follows.

The following symmetric linear demand system is assumed:

$$x = a - bp(1+t) + cq(1+t)$$
$$y = a - bq(1+t) + cp(1+t)$$

where $b > c > 0$ and for simplicity we set $a = b = 1$.

Net marginal revenue for firm 1 will be:

$$NMR_1 = [1 + cq(1+t)] - 2(1+t)p$$

---

6 $\partial_1 \phi_1 = \left[ P(\partial_1 x)^2 - (\partial_1 x + P\partial_1 x)x \right] / x^2$, will be positive if the demand function is concave or linear, but also if it is moderately convex. That is, as long as the marginal revenue curve is steeper than the demand curve, which implies $\partial_1 x + P\partial_1 x < 0$.

7 Strategic substitutability requires that an increase in the price of the rival creates a negative shock on marginal profitability $(\partial_1 x \pi_1 < 0)$. When marginal costs are constant this implies that an increase in the rival’s price increases own-price elasticity. Price Olen has to fall to restore the first order condition.

8 $\text{sign} \left[ \partial_1 \phi_1 + \partial_2 \phi_1 \right] = \text{sign} \left[ \partial_1 x(P\partial_1 x + \partial_2 x) - (\partial_1 x + P\partial_1 x)x - x\partial_2 x \right]$. The first two terms will be positive if the demand function is not too convex. $\partial_1 x \leq 0$ will therefore imply $\text{sign} \left[ \partial_1 \phi_1 + \partial_2 \phi_1 \right] < 0$.

9 The term in parenthesis in (3) would then be larger.

10 Note that, when the products are strategic complements, the negative impact on profitability is of a smaller magnitude since higher taxes on other goods tend to make demand more inelastic. This counteracts somewhat the initial tendency to lower prices. However, as stated in our text, strategic complementarity plays in the end an important role in the process of increasingly lower prices.
and marginal costs as a function of price are:

\[ MC = -k(1+t) \]

Note that, in equilibrium, both net marginal revenue and marginal cost are negative, since the latter can only be negative when expressed as a function of price. Figure 1 reflects a possible equilibrium price \( p^* \), given \( q^* \).

**Figure 1**

The discontinuous schedules reflect the new NMR and MC functions when the tax rate goes up and \( p^{**} \) is the new equilibrium price. At the old equilibrium price, marginal profits become negative, thus leading to a price reduction. The tax increase causes a decrease in marginal cost. However, for most demand systems, the net revenue function shifts to the left and becomes more concave and the decrease in net marginal revenue is larger, creating a negative profitability gap.

### 3.2. Market Prices

**Proposition 2.** An increase in the VAT rate will cause a proportionally smaller increase in final equilibrium prices if the demand functions are not too convex in their own price. When the products are strategic complements, we also require non-positive cross-partial derivatives of the demand functions.

This proposition can be easily derived from the previous results by looking at the first order conditions for this problem in (1).

If these conditions have to be satisfied when net prices fall, it must be the case that \( 1 - (1/\phi,) \) goes up, which implies an increase in \( \phi, \). Under the demand assumptions stated in the proposition, this elasticity will increase only if final prices increase.
Furthermore, it is obvious that this increase in market prices will be proportionally smaller than the increase in tax rates. Formally, from $P = p(l + t)$ and defining total elasticities as:

$$e = \frac{dP}{d(l + t)} \frac{P}{(l + t)} \quad \mu = \frac{dp}{d(l + t)} \frac{p}{(1 + t)}$$

we can easily derive the following relationship:

$$e = \mu + 1$$

Since $\mu < 0$, $|e| < 1$ as stated above.

A corollary of the previous propositions is that tax increases will decrease the volume of output for both goods and the net profitability of both firms.

4. Some Implications for Duopolists Competing in International Markets

4.1. Zero Transportation Costs

We consider next the case of two duopolists located in different countries (A, B), which do not charge the same VAT. The demand systems are assumed to be the same in both countries and, for the moment, we will assume that the goods are shipped to the foreign market at no cost. All other assumptions on technology and firm’s behavior are maintained. It is also assumed that arbitrage between markets is not feasible or, alternatively, that the arbitrage constraint is not binding. Then, Propositions 1 and 2 result in the following:

**Corollary 1:** Under the demand conditions stated in Proposition 1, the high-tax country will have higher final prices but lower net prices.

That is to say, the presence of a tax rate differential allows the firms to take advantage of the segmentation of the markets and it will be optimal for both firms to discriminate against consumers in the low-tax country.

Let us next consider the impact on welfare of the move towards an integrated market system. The change in welfare$^{11}$ for country A will be:

$$dW^A = (p^A - c) dX^A + (q^A - c) dX^B$$

and for B:

$$dW^B = (q^A - c) dY^A + (q^B - c) dY^B$$

Constraining the duopolists to charge a unique net price implies that the resulting equilibrium net prices will be between the unconstrained prices for markets A and B. That is: $(p^A, q^A) < (p, q) < (p^B, q^B)$.

Therefore, final prices will rise even more in the high-tax country (A), and fall in the low-tax one (B). Even though final prices for both goods go up, welfare for country A might increase as

$^{11}$ Total welfare for country A is: $w^A = U(X^A, y^A) - q^A y^A + (p^B - c) X^B - c X^A$. 

IESE Business School-University of Navarra - 7
long as the increase in exports to \(B\) equals or exceeds the decrease in domestic sales (since the surplus per unit is larger for the export good). Similarly, for country \(B\), welfare might also improve if increased domestic sales (\(y^B\)) equal or exceed decreased imports.

Overall, we will observe a welfare improvement as long as:

\[
(p^A - c)\, dX^A + (p^B - c)\, dX^B > 0 \quad 12
\]

which means that a welfare improvement can occur even when the increase in exports is outweighed by the fall in domestic sales. However, when home demand is very sensitive to price, and this is not the case for exports, we are likely to observe a decrease in welfare\(^{13}\).

Thus, if marginal costs are constant and transportation costs are negligible, tax rate differentials cause price discrimination and may act as a "hidden barrier to trade" as suggested by Geroski and Jacquemin (1985), except in the case where intra-industry trade falls with market integration. Even then, a welfare improvement for each of the countries is possible.

Next, we will explore the consequences of relaxing some of the cost assumptions maintained so far.

### 4.2. Positive Transportation Costs

Positive transportation costs can be incorporated into our model by looking at an asymmetric duopoly where the "domestic" firm (say firm 1, producing \(x\)) is the low-cost firm with marginal cost \(-k(1+t)\). Marginal cost for the foreign firm will then be: \((k+g)(1+t)\), where \(g\) is the unit transportation cost.

In previous sections, the symmetry of the problem was heavily used to obtain most of the results. However, the symmetry assumption is only instrumental in allowing analytical derivation. The continuity properties of the profit functions and the stability of equilibria seem to indicate that the previous propositions could be extended to the asymmetric case.

In addition we will have the following:

**Corollary 2:** When firms have different marginal costs, an increase in the VAT rate will decrease profitability of both the low-cost (domestic) and the high-cost (foreign) firm.

Note that this result will hold even when the reaction of the rival firm (to lower price) represents a positive shock on marginal profitability (strategic substitutability) since, in any case, both net prices end up falling. The corollary is in sharp contrast with the corresponding result for homogeneous goods obtained by Dierickx et al. (1986). They show that higher taxes can increase profits of domestic firms. In their model, the strategic effect can compensate the negative direct effect although this is usually not possible if all firms are identical (Quirmbach, 1986). The source of this profit increase (the positive shock on marginal profitability induced by the reaction of the rival) will not be present, in general, when products are differentiated and firms compete in prices. In our case both the direct effect \((\partial, \pi, d(1 + t))\), and the indirect

\[12\] Note that by symmetry \(dX^A = dy^A;\, dX^B = dy^B;\, p^A = q^A;\, p^B = q^B\).

\[13\] Related results have been obtained independently by Davidson et al. (1987).
effect will be negative since $\partial_3 \pi_1 < 0$ and $q$ falls while total net profits are positively related to $q \ (\partial_3 \pi_1 > 0)$.

This corollary can be easily shown in terms of Figure 2. Let $E$ be the equilibrium before the tax increase and let $x' = x(P,Q)$ and $y' = y(P,Q)$ be the isoquantity curves through that point. Since we know that net prices fall, we need only show that both quantities fall to ensure a decrease in profits. This is easily seen since the new tax-inclusive equilibrium will be in the region in price space $A$, where both quantities are smaller.

Finally, the next corollary shows the sense in which the intuition that a higher tax will benefit the domestic producer is true in this model.

**Figure 2**

**Corollary 3:** When firms have different marginal costs, an increase in the VAT rate will result in a bigger market share for low-cost (domestic) firms.

That is, even though total output and profitability fall, the competitive position of the domestic player is enhanced. Tax rate differentials usually hinder intra-industry trade (see above), and they can certainly make it harder for firms to compete effectively in foreign markets.

The corollary can be shown as follows. At an interior tax-free equilibrium $p<q$, and with the market share of the domestic firm $x/(x+y)$ between $5$ and $1$, the relationship between equilibrium net prices will hold for all possible interior equilibria which involve positive tax rates and will result in a similar relation between final prices.

Consider next the set of final prices $(P,Q)$ that make demand for any of the two goods equal to zero. This will be implicitly given by the prices that satisfy any of the following expressions: $0 = y(P,Q)$ and $0 = x(P,Q)$. If the demand system is symmetric it is easily shown that for the prices satisfying the first restriction $P>Q$. Similarly, $P<Q$ for the second one and $P=Q$ if both restrictions are satisfied. Consider next an equilibrium with a high tax rate such that demand for one of the goods becomes arbitrarily close to zero. Since it must be the case that $P<Q$, it must be the foreign good that is barely demanded. Thus, its market share tends to zero and the
market share of the domestic firm tends to one with higher taxes. This kind of outcome is illustrated for the linear model in Figure 3.

**Figure 3**

![Diagram showing the impact of changing VAT rates when marginal costs are not constant.]

### 5. The Impact of Changing VAT Rates when Marginal Costs Are not Constant

When marginal costs are not constant, expression (4) in section 3 has to be modified as follows:

\[
\partial_1 \pi_i = \left[1 + t \partial_i x \right] \left[ p \partial_{(1+1)} \phi_i (P, Q) / \phi_i^2 \right] - \partial_1 c \left( p^0 x + q \partial_i x \right)
\]

where \( \partial_1 c \) is the second derivative of the cost function \( c(x(P, Q)) \). Proposition 1 will hold if marginal costs are increasing \( (\partial_1 c > 0) \) but need not hold if marginal costs fall with output.

When marginal costs are increasing as a function of output, an increase in final prices will correspond to a decrease in output and marginal costs. Even if elasticity is held constant, a declining net price has to be matched by higher final prices, for the following first-order condition to be satisfied:

\[
p \left[1 - \left( p \phi_i (P, Q) \right) \right] = \partial_1 c(x(P, Q))
\]

This increase in final prices will be even larger if, as assumed earlier, elasticity increases with higher prices.

However, with increasing marginal costs, Propositions 1 and 2 cannot be readily extended to a multimarket framework since the markets are then interrelated from the supply side of the
model. An increase in the tax rate in one country will represent a positive shock in the other market, thus pushing net prices upwards in the low-tax country. This would seem to reinforce the tendency to have lower net prices in the high-tax country. Furthermore, the negative domestic shock tends to increase the market share at home and the positive shock abroad tends to favor the competitive position in the foreign market. However, the impact on profitability and IIT is difficult to ascertain in this case since lower output and profits at home have to be matched to increased sales and profits abroad.
Appendix
The "Ad Valorem" Model

The first order condition for firm 1 in this model is the following:

\[(A.1) \quad (1-t) \left[ x(P, Q) + P \partial_1 x \right] - k \partial_1 x = 0 \]

We are, again, interested in the sign of \( \partial_{13} \pi_1 \). From (A.1) we obtain:

\[
\partial_{13} \pi_1 = -\left[ x(P, Q) + P \partial_1 x \right] = -\left[ k \partial_1 x / (1-t) \right]
\]

The sign of \( \partial_{13} \pi_1 \) will be definitely positive in this case, which implies that final, tax-inclusive prices will go up. In contrast with the surcharge model, here we have a positive shock on marginal profitability. Note that, as in that model, in equilibrium both net marginal revenue and marginal costs as functions of prices are negative. The shock here will not affect the marginal cost schedule but it will make the net marginal revenue more concave.\(^{14}\) As a result, net marginal revenue increases, thus creating a positive profitability gap which pushes prices up. This initial shock is reinforced by upward sloping reaction functions.

The linear example that follows may clarify the source of this positive marginal profitability shock. The demand system is the same as was assumed in the text. The resulting net marginal revenue and marginal cost functions of firm 1 are: \( NMR_1 = (1-t) \left( 1-2p+cx \right) \); \( MC_1 = -k \)

These are depicted in Figure 4. The discontinuous schedule reflects the new net marginal revenue function. Its rotation increases marginal profitability (the segment AB in the figure), and leads to a move from \( p^* \) to \( p^{**} \).

Figure 4

The question arises as to what will be the impact of higher taxes on net prices, defined in this instance as \( p = (1-t) P \) and \( q = (1-t) Q \).

\(^{14}\) \( \partial^2 NMR / \partial P \partial_1 x (1-t) = 2 \partial_1 x + p \partial_{11} x < 0 \). Note that when marginal costs are zero this \textit{ad-valorem} tax does not affect the optimal choice of the firm.
Appendix (continued)

A standard transformation of (A.1) yields the usual elasticity expression:

\[ p \left[ 1 - \frac{1}{\phi_1(P,Q)} \right] = k \]

If the own-price elasticity increases with higher prices (the assumption required for propositions 1 and 2 in the text), net prices will have to fall for the first-order conditions to be satisfied.

Finally, note that with this specification final prices will also increase proportionally less than the increase in tax rates (in fact than the decrease in \((1-t)\)). From the relation between net and tax-inclusive prices \((p=(1-t)P)\) we obtain:

\[ e = \mu + 1 \]

where:

\[ e = \left[ \frac{dP/d(1 + t)}{P/(1 + t)} \right] \quad \mu = \left[ \frac{dp/d(1 + t)}{p/(1 + t)} \right] \]

Since \(\mu > 0\), (recall that \(dp/dt < 0\)), and \(e < 0\), we obtain \(e + 1 > 0\) which implies \(|e| < 1\).
References


