ON MANAGERIAL CONTRACTING WITH ASYMMETRIC INFORMATION

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Abstract
In Holmstrom and Ricart [1986], we presented a formal model of an incongruity in risk preferences between the manager and the owners of a firm based on learning about managerial talent. We also explored the nature of an optimal incentive contract in the case where the manager may withhold but not misrepresent information about investment returns.

In this paper, I study a model where only simple 0-1 investment decisions are considered. I analyze the effect on the optimal contracts of two different kinds of asymmetries in information. One is the labor market asymmetry where the current employer is better informed on the manager’s performance than potential alternative employers. It is shown that this asymmetry acts much in the same way as any firm-specific human capital and that optimal contracts may include firings when the manager is unable to exploit his private information. Furthermore, the asymmetric case may be preferred ex ante for young managers over the symmetric counterpart.

The second asymmetry I study is generated when the manager’s reports are unverifiable and therefore the manager can misrepresent his information. This asymmetry introduces additional variation in the wage scheme but maintains the same qualitative properties in expectation, in a way similar to the standard moral hazard characterization. Finally, I link these asymmetries and discuss the introduction of forecasting ability in the model and other extensions.

The integration of the ideas in this paper with well-known results in contractual theory allows us to understand the form of optimal wage contracts in different information structures.

Comments are welcome

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1. Introduction

It is well known in the literature on labor contracts that, with symmetric information and enforceable contracts, the best contractual arrangement between a risk neutral firm and a risk averse worker is a constant wage across periods and states of nature. As it is done in Ito [1984], one can interpret such a contract as a combination of a spot wage determination (i.e. wage equal marginal product of labor) plus an insurance contract offered by the risk neutral firm to the worker. The worker’s performance is a valuable piece of information which can be used to update the firm’s assessment about the worker’s ability or talent. Therefore, we learn about his characteristics from his performance. If all this information is public, it will be difficult to keep the good workers into the firm with a constant wage. If the worker’s reputation has been enhanced, and he can costlessly prove it, he will obtain wage increases (reflecting his enhanced human capital) by changing firms. Since slavery is illegal, it is very difficult to enforce the original contract.

Harris and Holmstrom [1982] studied this situation characterized by symmetric information, indenture prohibited. They prove that wages are downward rigid and tight (or upward rigid) in the sense that they do not decrease over time and increases take place only whenever the worker’s market value is above his current wage.

We can decompose the rigid contract as follows: the risk-neutral firm insures the risk-averse worker against the worst situation by assuring him that his income will never decrease below his current wage. So, the insured wage for period $t+1$ is $T_{t+1} = w_t$, where $w_t$ is the wage at time $t$. But, since indenture is prohibited, the worker cannot be fully insured (as used to be done) and his wage must rise whenever his market value increases above the insured wage. This possibility decreases the level of the insured wage which will be below the optimal constant wage with enforceable contracts.

In conclusion, the Harris and Holmstrom solution can be schematically represented as:

\[ w_{t+1} = \max \{ T_{t+1}, x_{t+1} \} \]

\[ T_{t+1} = w_t \]  \hspace{1cm} (a)

\[ x_{t+1} = \text{market value at } t+1. \]
Keeping the information symmetry, one can try to introduce some decision making in the Harris and Holmstrom setup. This has been done in Holmstrom and Ricart i Costa [1986] (referred to as HRC). Section 2 will cover in detail some of the results from HRC as a benchmark reference to the new results in the paper. In that paper, we introduced some degree of managerial decision-making but we maintained the symmetric information assumption. (In the rest of the paper, I will use the term manager instead of worker).

Looking at the wage part of the contract, one can say that HRC replicate the results mentioned above. Therefore, with enforceable contracts, the wage is constant across time but, when indenture is prohibited and information is symmetric at any point of time (we call this the public signal case), the wage contract has the same form as the one in (a), except that the decision process makes calculation of the actual wage somewhat different and utility-dependent.

If we allow the manager to obtain some private information (private signal case), verifiable once presented to the firm (therefore restoring information symmetry ex post), then the wage contract is slightly modified by adding, for some signals, a bribe value above the insured wage to induce the manager to make "the right decision". This wage schedule can be represented by

\[ w_{t+1} = \max \{T_{t+1}, x_{t+1}\} \]
\[ T_{t+1} = \max \{w_0, b_t(s_t)\} \]  \hspace{1cm} (b)

\( b_t(s_t) \) = bribe value to induce the right decision at \( t \), after observing the signal \( s_t \).

Note that the only difference between (a) and (b) is contained in the insured value \( T \) and does not come from the introduction of decision-making but from the temporal asymmetry in information (from the time of observation to that of verification of the signal). Another interesting question refers to the way in which decisions are affected in this situation. In HRC, we proved that the decision should take into consideration the discounted present value of three components: financial returns, human value returns (gains from learning), and the cost of wage variability due to the manager’s risk aversion.

In this paper, I would like to take a further step and, using a simpler but robust model, study the effects of asymmetric information. I start (Section 3) by studying the effect of asymmetries in the labor market when the current employer is better informed about the manager than the alternative employers. Specifically, outside employers cannot verify or observe the manager’s signal. I prove that the wage contract does not differ much from the basic Harris-Holmstrom reference (a). The only difference arises in relation to the manager’s market value.

I distinguish two cases. First, suppose that the manager has the same information as the firm and can exploit it. I claim that the market will be able to separate the different types of manager, but, since this separation is costly, the manager’s market value will be below his actual productivity, allowing the incumbent firm to collect positive quasi-rents. Hence, the information differential acts in the same way as the traditional firm-specific human capital that has been studied so extensively in the labor contracts literature. Furthermore, the asymmetric case is ex ante preferred by young managers.

Second, suppose that the manager cannot exploit his information (perhaps the firm is better informed than himself). I develop a model that resembles Greenwald’s model [1986] but with long-term contracts, where I prove that some amount of "ex post involuntary firings" will be included in the optimal contract. For this "pooling" case, I obtain small deviations from the
basic rigid contracts given in (a), because a fired manager will experiment a small decrease in his wage income.

Finally, I study alternative solutions to the extreme cases exposed above. The applicability of the different alternatives depends on the assumptions on the wage contracts' observability and timing. We show how important it is to determine the different parties' global information structure and bargaining behavior in determining optimal contracts.

So far, I have mentioned asymmetries in the labor market or external asymmetries, but what about the effect of internal asymmetries? What will happen if the manager is better informed than the firm or has no verifiable information that is useful for the decision?

Rogerson [1985] studies some properties of contracts in a repeated moral hazard situation. The manager's choice of effort is not observable and the output is used as a surrogate measure of effort. He uses enforceable contracts and, therefore, his results are best compared to the constant wage solution given at the beginning of this introduction. To put Rogerson's model in our contractual framework, we can talk about finding the optimal enforceable contract when the manager's information is unverifiable. Since the level of output will depend on the manager's talent as well as the characteristics of the project, the firm is unable to distinguish between a talented manager with a difficult project and a non-talented manager with an easy project.

To obtain a truthful revelation, a constant wage is unfeasible and a contract with the same characteristics as Rogerson's will be optimal. The wage schedule does not give an intertemporal marginal rate of substitution of one (as before), and, instead, this rate is one only in expectation. That is:

\[ E[u'(w_{t+1})/u'(w_t) \mid \text{information at } t] = 1. \]  \hspace{1cm} (c)

This wage contract will be used as the new benchmark for the unverifiable signal case, present whenever the employer cannot verify or observe the manager's signal. In Section 4, I show that, once indenture is prohibited, the contractual solution is a combination of (a) and (c). That is, there exists a contingent insured value \( T \) such that:

\[ E[u'(T_{t+1})/u'(w_t) \mid \text{information at } t] = 1 \]

and the actual wage, \( w_{t+1} \), is above \( T_{t+1} \) only when the market value is greater than the insured value \( T_{t+1} \), i.e. \( w_{t+1} = \max \{T_{t+1}, x_{t+1}\} \).

In summary, one can easily observe the effect of each restriction on contract determination. Essentially, internal asymmetries in information affect contracts, transforming the marginal rate of substitution, in insured terms, to one only in expectations and in a particular fashion. Additionally, once indenture is prohibited, we obtain a downward rigid (or similar) structure where market pressures bid the wages of successful managers upwards. The market value is determined by the information available to the labor market and by the manager's capacity to exploit his information. Additionally, I show that the qualitative results obtained in HRC referring to the decision-making process are essentially unaffected by changes in information.

I conclude the paper by collapsing both asymmetries together in order to study situations where the manager's unknown characteristics can be interpreted as his forecasting ability (which requires the unverifiability of his reports). This is done in Section 5.
There follows a graphical illustration that integrates the different information structures discussed above, with references to the corresponding sections in the paper.

### LEARNING MODELS OF REPUTATION
- Harris and Holmstrom (1982): SYMMETRIC INFORMATION NO DECISION MAKING
- Holsmtrom and Ricart i Costa (1986) SYMMETRIC INFORMATION MANAGERIAL DECISION MAKING (Section 2)
- ASYMMETRIC INFORMATION INVESTMENT DECISIONS (Section 3)
  - SEPARABLE (Section 3.1)
  - POOLING (Section 3.2)

### MORAL HAZARD
- Rogerson (1985) EFFORT CHOICE ENFORCEABLE CONTRACTS
- UNVERIFIABLE REPORTS INDENTURE PROHIBITED (Section 4)
- GENERAL FORECASTING MODEL (Section 5)

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**2. The Model**

There are two periods, \( t = 1, 2 \), technologically identical and independent. In each period, a publicly observed decision \( a(t) \in \{0, 1\} \) is taken, where 1 represents accepting while 0 represents rejecting a prospective investment. If the investment is accepted, it yields a net payoff of

\[
y(t) = y(\eta, \theta_t) \in Y, \tag{1}
\]

where \( \eta \) is a quantified measure of managerial ability and \( \theta_t \) is a random state of nature. Notice that ability does not change between periods though beliefs about ability generally will. There is always the opportunity of rejecting the investment, which yields a net payoff of zero.

The owner is risk-neutral and the manager is risk-averse with a temporal utility function over consumption given by

\[
u(c_1, c_2) = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1. \tag{2}
\]

Both parties use the same discount value \( \beta \). The manager cannot borrow or save and will therefore consume his income in each period.

The manager observes a signal, \( s(t) \in S \), about the payoff prospects before \( a(t) \) must be determined. I will assume that both the payoff set \( Y \) and the signal set \( S \) are finite. That is
\( y(t) \in \{y_1, y_2, ..., y_n\}, \) where \( y_1 < y_2 < ... < y_n \), and \( s(t) \in \{s_1, s_2, ..., s_m\}. \)

The manager and the owner share beliefs (at least initially) about ability, signals, and states. These beliefs are encoded in the following probability assessments:

\[
\begin{align*}
p^{(1)}(\eta) & - \text{the prior distribution of } \eta, \\
r = (r_1, r_2, ..., r_m), & \text{ where } r_i = Pr(s_i), \\
m(\eta) &= (m_{ij}(\eta); i = 1, ..., m, j = 1, ..., n), \text{ where } m_{ij}(\eta) = Pr(y_j \mid s_i, \eta).
\end{align*}
\]

This list does not include all the primitive assessments for notational simplicity. I note, however, the following. Ability and state may be dependent. The signal and state are dependent. But the signal and ability are independent. This last assumption will be relaxed later but it simplifies our analysis. This implies that ability has a purely productive function- it is a measure of management skills rather than forecasting skills\(^1\).

For notational convenience, I define:

\[
m_{ij}(p) = \int m_{ij}(\eta) \ p(\eta) \ d\eta,
\]

and I will use \( m_{ij} \) when it refers to \( m_{ij}(p^{(1)}) \), with \( p^{(1)} \) being the prior distribution on the manager’s ability. I will also assume:

\[
ri > 0 ; \ m_{ij} > 0 \ \forall p, \ i = 1, ..., m, j = 1, ..., n. \quad (A1)
\]

There is an outside labor market in which the manager can find alternative employment. For simplicity, I assume that this market has the same technological opportunities as the firm in which the manager is presently employed.

The particular sequence of events and the information that each party is able to observe will vary in each section and I postpone the exposition to each particular case.

The key feature of our model is learning about ability. When an investment is accepted in the first period, the distribution over outputs may be used to update beliefs about ability. In statistical language, we have an experiment described by \( \{m_{ij}(\eta)\} \) with \( \eta \) as the parameter.

Let \( p^{(2)} \) be the posterior ability distribution which, as a function of output, is itself random. By applying Bayes’ rule, the stochastic process taking priors to posteriors forms a martingale, i.e.

\[
E[p^{(2)} \mid s(1)] = p^{(1)},
\]

where the expectation is conditional on the first-period signal. Let

\[
Y(s_i, p) = \sum y_j m_{ij}(p);
\]

this represents the expected value of output given that the investment is accepted, a signal \( s_i \) and a manager whose ability is assessed by \( p \).

\(^1\) A forecasting interpretation requires the signal to be dependent on ability, so that talented managers get "better signals" on investment possibilities. However, since the signal is privately observed, its dependence on ability implies that the manager can update his beliefs before taking his decision. This fact generates an asymmetry of information and it is better addressed, because of its very nature, by an unverifiable reports model. Section 5 will deal with this point.
Let \( \alpha : S \rightarrow \{0, 1\} \) be a decision rule describing the action chosen for each signal. I represent \( \alpha = (\alpha_1, \ldots, \alpha_m) \).

The decision rule that maximizes expected output is given by

\[
\alpha^*_i(p) = 1 \text{ if and only if } Y(s_i, p) \geq 0. \tag{4}
\]

With this decision rule, the value of the manager with ability distribution \( p \) is given by:

\[
z(p) = \sum_i r_i \max\{Y(s_i, p), 0\}, \tag{5}
\]

which is a convex function of \( p \).

Let the manager’s marginal product in period \( t \) be \( z_t \). It depends, of course, on the decision rule and \( p^{(t)} \). By (4) we will always have \( z_t \leq z(p^{(t)}) \). Combining the convexity of \( z() \) with the martingale property (3) we have (by Jensen’s inequality):

**Lemma:** The expected value of the manager’s marginal product in the second period, conditional on any signal \( s(1) \), satisfies:

\[
E[ z(p^{(2)}) \mid s(1) ] \geq z(p^{(1)}). \tag{6}
\]

This lemma states the obvious fact that learning has value in general. Define the discounted value of learning for a given signal as \( L(s(1), p^{(1)}) = \beta (E[ z(p^{(2)}) \mid s(1) ] - z(1)) \)

**Remark:** We can generalize our model by allowing the ability to change between periods. To simplify the exposition, I choose to concentrate on the pure learning model where ability is unchanged.

A contract \( \delta \) is a pair \((\omega, \alpha)\), where \( \omega = (w_1(h_1), w_2(h_1, h_2)) \) is the wage contract contingent on the observed history, and \( \alpha \) stands for the decision rules.

In HRC, we characterized the optimal contracts for the symmetric information case. This is the case characterized by the following sequence of events within each (identical) period: first, the manager is paid a wage, which he consumes instantly (the results here do not depend on the instant of the period when he is paid). This wage need not be his marginal product in the period since he could be on a contract. Next the manager observes a signal, which he either reports truthfully or withholds from reporting. The owner decides what action to take. If the investment is accepted, the output is realized, and all parties observe the signal, the action, and the outcome.

In the symmetric information case, we obtain the following results:

**Proposition 1:** The optimal contract for the symmetric information case, without indenture, and privately observed signals is \( \delta^* = (w_1, w_2, \alpha^1, \alpha^2) \) such that:

(a) \( w_2(h_1) = \max \{w_1, b_i, z_2\}, \forall h_1 = (y(1), a(1), s_1) \text{ with } a(1) = 1, \)

where \( z_2 = z(p^{(2)}(h_1)) \), \( w_1 \) is a constant first-period wage, and \( b_i \) is a bribe value given by

\[
E[u(\max \{ z_2, b_i \} \mid s_1)] = u(z(p^{(1)})) \tag{7}
\]

and if (7) has no solution, then \( b_i = -\infty \). If the investment is not accepted \( (a(1) = 0) \), then there is no learning and \( w_2(h_1) = z(p^{(1)}) \).
\[ \alpha' = 1 \text{ if and only if } Y(s_i, p^{(i)}) + L(s_i, p^{(i)}) + H(w_i, s_i) \geq 0 \text{ and } \alpha^2 = \alpha^* , \text{ where } H(w_i, s_i) \text{ is a risk term proportional to the manager’s risk aversion.} \]

**Proof:** Apply Proposition 2 from HRC.

**Remark:** The solution for the public signal case is the same solution with \( b_i = -\infty \) for all signals (see HRC).

Furthermore, we have:

**Corollary 1:** For a risk-averse manager, \( H(w_i, s_i) < 0 \). That is, the hurdle rate for the expected value of total capital (financial gains plus learning: \( Y(s, p) + L(s, p) \)) is above the market cost of capital in selecting investment projects.

**Proof:** See Proposition 3 in HRC.

### 3. Asymmetry in the Managerial Labor Market

Let us start by illustrating the effect of having asymmetric information in the labor market before the introduction of contracts. To this end, suppose that we have only two signals, \( s(t) \in \{0, 1\} \), with equal probability. Output is given by \( y(t) = s(t) + e(t) \), where \( e(t) \) can be 0 or 1 with \( p(e(t) = 1) = \eta \), where \( \eta \) is a variable in \([0, 1]\) that quantifies the manager’s talent. If \( s(1) \) is observed, the manager is exposed to a Bernoulli experiment every time that a project is accepted. Let \( w_+ \) and \( w_- \) represent his marginal value if he succeeds \( (e(1) = 1) \) or fails \( (e(1) = 0) \), respectively. Obviously, \( w_+ > w > w_- \) where \( w \) is his expected value ex ante, before the experiment takes place.

Now, assume that \( s(1) \) cannot be observed by the outside employers. To center the discussion, suppose that the observed output is \( y(1) = 1 \). The outside employers cannot verify the signal \( s(t) \) and, therefore, cannot distinguish between a talented manager who observed \( s(1) = 0 \) and a non-talented manager who observed \( s(1) = 1 \).

An alternative employer can offer a pooling wage \( w \) to any manager in the second period. But in this situation, the talented managers will stay with the current employer (since the owner knows that their value is \( w_+ > w \)) while the non-talented managers (with value \( w_- < w \)) will accept the alternative offer. And the outside employers will attract only the lemons. Therefore, in equilibrium, the outside market will only offer the lower wage \( w_- \).

The above argument introduces a serious adverse selection problem to the talented manager who will be unable to collect the rents of his enhanced human capital. Greenwald [1980] studies more extensively the effect of this adverse selection in a model where only spot contracting – period by period – is considered.

In this section, I study the effect of introducing contractual arrangements in this kind of situation. To make things simpler, I assume that \( S = \{s_1, s_2\} \) and that signals are verifiable by the current employer but not by potential outside employers. Hence, the asymmetry is only present in the labor market. The actual meaning of the information not observed by the market is not essential. The relevant point here is that the current employer and the manager have superior information.
The manager’s wage is paid at the end of each period and therefore can be contingent on any information the firm has gathered during the period. This fact is essential as a tool for the manager to separate himself from lower ability individuals. The first-period employer offers a two-period contract to the manager. The manager is free to seek alternative employment in the second period because indenture is prohibited.

To further simplify our model while retaining the essential elements, I assume that the decision rule in the first period specifies that investments will be accepted for both signals. Otherwise, since the decision is observable, the market would be able to infer perfectly the signal from the decision, and no asymmetry would be present.

One important question which is postponed until section 3.4 relates to the observability of the wage contracts. For the analysis of the next section, I implicitly assume that while wage contracts are observable, actual wages are not observable until they are paid.

### 3.2 Separable Solution

For any given outcome $y_j$ in the first period, the manager could have observed either signal $s_1$ or $s_2$. While the manager and the firm know this information and are able to update their beliefs about the manager's ability, the outside employer can only assess that the manager's beliefs $p_{i[s]}$ are either $p_{i_1}$ or $p_{i_2}$ with certain probabilities $q_{i_1}$, $q_{i_2}$. Note that if ability changes from period to period, our arguments are unaffected by the particular way we obtain $p_{i_1}$, $i = 1, 2$. I will refer to $p_{i_1}$ as the manager type that produced $y_j$ in the first period. I will indicate by $p$ the common knowledge prior to $p_{i[s]}$.

In what follows I suppress the superindex $j$ wherever it is clear. Also, without loss of generality, I assume that the marginal product of type 2 is higher than that of type 1, i.e., $z(p_2) > z(p_1)$. Finally, I recall from (5) that the manager's value is given by

$$z(p_i) = \sum_k r_k \max \{0, \sum \alpha \cdot \text{mke}(p_i)\}, \quad i = 1, 2$$

Suppose that the employer offers a pooling contract for any manager with output record $y_j$. That is, a common contract is offered to both types. The best pooling contract in this situation is a constant wage equal to the manager's expected value. Since the manager is better informed, the investment decision is delegated to him. Furthermore, he does not have any reason to deviate from the efficient decision $\alpha'(p_i)$. Therefore, the pooling wage is only dependent on the observed output $y_j$, and is given by:

$$w_q = q_1 z(p_1) + q_2 z(p_2).$$

A type 2 manager will not be satisfied with this offer, because he is paid below his productivity. He can go to another employer (or the same one for that matter) and, with the pooling offer in hand, ask for an alternative better offer, which would be unacceptable if he were of type 1 and would give positive profits to the employer. Such an offer exists and will be agreed upon between the manager and the new employer. Therefore, the pooling contract will be acceptable only to the low types, and any firm offering it will end up losing money.

Of course, this is not a new argument, since it has been used previously in the literature (see, for instance, Riley [1979], Kreps [1984] or Ricart and Costa [1984]). The main point is that the manager's ability to exploit his private information will force the market to offer two different contracts and give the manager a choice. Clearly, the contracts must be designed in a way that
each type of manager chooses the contract designed for him. The main element which will allow this separation is that higher types are better suited for the job and may accept some amount of risk unacceptable to the lower type.

Let us now characterize the proposed solution. First of all, observe that the low-type manager may always claim to be low and obtain at least a flat wage $z(p_1)$. Since the contract must be safe – zero expected profits given all available information – $z(p_1)$ will be the low type’s market value. Therefore he will be offered the following contract:

$$\delta_1 = (w^1, \alpha^*(p_1)),$$

where $w^1 = z(p_1)$.

The high-ability manager will be offered an alternative contract $\delta_2$. This contract must maximize his expected utility over all safe contracts that are protected from the low type’s deviations. Since the element that allows separation is risk aversion, the contract should be contingent on the second period history $h_2 = (s_k, y_e, a) \in S x Y x A$. Therefore, the high-type contract is given by:

$$\delta_2 = (W, \alpha),$$

where $W = (w_{ke} \text{ if } a = 1, v_k \text{ if } a = 0)$, $k = 1, 2$, $e = 1, \ldots, n$,

such that it is the solution of the following program:

$$\begin{align*}
\text{Max} & \sum_k r_k [\alpha_k \sum_e u(w_{ke}) m_{ke}(p_2) + (1 - \alpha_k) u(v_k)], \\
\text{s.t.} & \sum_k r_k [\alpha_k \sum_e (y_e - w_{ke}) m_{ke}(p_2) - (1 - \alpha_k) v_k] \geq 0, \quad (P1) \\
& u(z(p_1)) \geq \sum_k r_k [\alpha_k \sum_e u(w_{ke}) m_{ke}(p_1) + (1 - \alpha_k) u(v_k)].
\end{align*}$$

The contract maximizes the expected utility of the type 2 manager subject to non-negative profits to the firm and assuring that type 1 managers will not deviate and choose $\delta_2$. Then we have:

**Proposition 2:** The optimal contract $\delta_2$ is characterized by

- (a) $v_k = w$ independent of $k$, where $u'(w) = \gamma/(1 - \mu)$;
- (b) $w_{ke}$ satisfies $1/u'(w_{ke}) = (1 - \mu)/(\gamma + (\mu/\gamma) m_{ke}(p_2)/m_{ke}(p_1))$,

where $\gamma > 0, 0 < \mu < 1$ are Lagrange multipliers for constraints (i) and (ii);

- (c) $\alpha_k = 1$ if and only if $\sum_e m_{ke}(p_2) [y_e + H_{ke}] - H_{k0} \geq 0$, where

$$H_{ke} = (u(w_{ke}) - u'(w_{ke})w_{ke})/u'(w_{ke})$$

(use $v_k$ in $H_{k0}$).

**Proof:** (See Ricart i Costa [1986]). The proof is an application of Kuhn-Tucker conditions and some additional elaboration to obtain (c).

Let $x_i$ represent the market value (second period) of a manager of type $i$, $i = 1, 2$. This enables the following corollary to be easily derived, with the qualitative results we are interested in:

**Corollary 2:**

- (a) $x_1 = z(p_1)$,
- (b) $x_1 \leq x_2 < z(p_2)$ for a risk averse manager.
Proof: Only (b) needs some explanation. \( x_1 \leq x_2 \) since \( v_k = z(p_1) \) is always feasible in (P1). Normally we expect to do better and obtain \( x_1 < x_2 \). Finally, since the manager is risk-averse and separation is costly (he has to accept risk), we must have \( x_2 < z(p_2) \). That is, the certainty equivalent value is below the mean value for a risk-averse manager. Q.E.D.

The decision rule for the high-ability manager is not relevant to my analysis. Still, it is interesting to note that it may differ from \( \alpha^*(p_2) \), since now we have to consider also the risk factor \( \text{He}_e \).

With the market value function for each output \( j \) at hand, I can now approach the characterization of the long-term contract. The contract specifies the wage for both periods contingent on past history. We do not need to consider the decision rule in the contract. The first-period decision rule is assumed to be “invest always”, while the second period decision rule will be first-best as will become clear later on. The contract is specified by:

\[
\delta^L = (w_1(h_1), w_2(h_1, h_2))
\]

where \( h_t = (s(t), y(t)) \) is the history in period \( t \) (a(t) is fixed). Let \( x(h_1) \) represent the market value of a manager with history \( h_1 \). Then the optimal long-term contract is the solution of:

Max \( E[u(w_1(h_1)) + \beta u(w_2(h_1, h_2))] \),

s.t. (i) \( E[y(1) - w_1(h_1) + \beta (y(2) - w_2(h_1, h_2))] \geq 0 \),

(ii) \( w_2(h_1, h_2) \geq x(h_1), \forall h_1, h_2 \).

where (i) is a zero-profits condition and (ii) are the market constraints that assure that the manager will not be bid away in the second period. The solution of (P2) is given in the following proposition.

**Proposition 3:** The optimal long-term contract is given by

(a) \( w_1(h_1) = w, \text{ independent of } h_1; \)

(b) \( w_2(h_1, h_2) = w_2(h_1) = \max\{w, x(h_1)\}, \text{ independent of } h_2, \text{ where } w \text{ solves (I use } x_{ij} = x(h_1) \text{ if } h_1 = (s_i, y_j)) \)

\[
\sum_i r_i \sum_j m_{ij} [y_j - w + z(p_i) - \max\{w, x_{ij}\}] = 0.
\]

Remark: Since \( w_2 \) is not contingent on the second period output, there is no incentive to deviate from the optimal decision rule \( \alpha^*(p_i) \).

**Proof:** It is a simple application of Kuhn-Tucker conditions. The first-order conditions are:

\[
\lambda + \gamma(h_1, h_2) = 0.
\]

\[
u'(w_2(h_1, h_2)) \lambda + \gamma(h_1, h_2) = 0.
\]

Therefore our solution with \( \lambda = u'(w) \) and \( \gamma(h_1, h_2) = u'(w) - u'(w_2(h_1)) \) satisfies the F.O.C. and the complementary slackness conditions. Thus, the solution (of a concave program) is optimal. Q.E.D.

The solution I have obtained is essentially the same as that for the symmetric information case when indenture is prohibited but the signal is public. The only difference is the determination of \( x_{ij} \). In the symmetric information case, \( x_{ij} = z(p_i) \), and the second-period wage for high-
ability managers equals their productivity. In the case here, the wage is also associated with ability, but because communicating information about ability is costly, the market value and hence the wage is below productivity (for high ability). Thus, the asymmetry in information acts in the same way as any type of firm-specific human capital. The effect of the information about ability is to increase the manager’s value for the current employer more than for the outside employer, allowing the firm to collect quasi-rents from high-ability managers in the second period. Of course, this will increase the initial wage level \( w \) above that in the symmetric information case.

This last observation brings us to an important result in this section. Without contracts, the asymmetry in the market seriously jeopardizes the high-ability managers, as we saw in the introductory example. They cannot obtain the full amount of their productivity from the market. This is still true once contracts are introduced. However, by increasing his guaranteed wage \( w_1 \), the manager is able to capture the quasi-rents the firm may obtain in the second period. Furthermore, since the solution for the symmetric case is feasible in (P2), the young manager is ex ante better off in the asymmetric case than he was in the symmetric counterpart. Therefore, the second-period adverse selection problem ends up being ex ante beneficial for young managers since it allows them to choose a more uniform consumption stream.

Ex-post, the high-ability manager will clearly prefer the symmetric solution but he is unable to obtain his full product because communication is costly. But ex ante, he prefers the asymmetric contract and therefore, even in situations where the information may be verifiable, it is in the young manager’s interest to agree upon a contract that legally forces him to keep inside information secret. An important point here is that the manager recovers his ex-post loss if he is talented, with an increase in his first-period wage. Furthermore, in the equilibrium he never changes jobs and therefore, there is no loss in efficiency, i.e. his second-period wage in the current employer is not dependent on his outcome in the second period.

3.3 A Pooling Solution

I also want to illustrate the situation where only pooling contracts are offered, i.e., the situation where the manager does not exploit his private information. Later on, I will present a more general discussion of alternatives. This will bring my model closer to Greenwald [1986]. One possible reason why pooling contracts may be offered is that the manager does not have any private information. We can imagine that only the current employer observes the output. Another explanation is that wages must be paid at the beginning of the period and the manager cannot use his information to separate himself from lower types. This last assumption is the one I use below. The model is the same as that presented above but with wages payable at the beginning of the period. Furthermore, I assume that the manager’s decision about continuing his employment is as follows. He will quit if the market offer exceeds his current wage. In addition, if the current employer matches or exceeds the market offer, the manager may still quit with probability \( t \). This is exactly the quit behavior assumed in Greenwald [1986]. For simplicity, I do not discount.

I need some additional notation. Let \( b_{ij} \) represent the probability that a manager, with history \((s_i, y_j)\) in the first period, leaves his current employment (either voluntarily or because he is fired). Since there are only two signals, I refer to them as \( s_i \) and \( s_k \). For each \( j \), I assume without loss of generality that \( z(p_{ik}) > z(p_{ij}) \). Note that \( k \) and \( i \) may depend on \( j \); i.e., they can change as \( j \) changes, but one will always be higher than the other. As before, I let \( q_i^j \) and \( q_k^j \) be the prior probabilities of each type, i
or k, before any b_{ij} is considered. By Bayes’ rule, given that managers of type i leave their employment with probability b_{ij} and managers of type k leave their employment only with probability \( \mu \), the posterior probabilities of each type are defined by:

\[
\begin{align*}
d_{ij} &= b_{ij} q_{ij} / (b_{ij} q_{ij} + \mu q_k) \\
d_{ij} &= 1 - d_{ij} \\
d_i &= (d_{ij}, d_k)
\end{align*}
\]

If the outside employer’s beliefs are represented by d_j, he will offer a pooling contract given by:

\[
W_p(d^i) = d_{ij} z(p_i^j) + d_k z(p_k^j).
\]

Finally, define t^i as in (10) when b_{ij} = 1, and note that when b_{ij} = \( \mu \), then d^i = q^i = (q_i^j, q_k^j).

Given these assumptions we have that b_{ij} \in [\mu, 1] and therefore d_{ij} \in [t^i, q^i]. With this notation, one can easily prove the following inequalities:

\[
z(p_i^j) < w_p(t^j) \leq w_p(d^j) \leq w_p(q^j) < z(p_k^j)
\]

Now suppose that market beliefs are represented by d^i; therefore, market offers will be represented by w_p(d^j). It follows that a long-term contract, agreed between the manager and the first employer, is characterized by

\[
\delta_p = \{ w, (w_{ij}, b_{ij}); i = 1, 2, j = 1,..., n \},
\]

where w and w_{ij} are the wages and bid the probabilities of continuing employment. The optimal contract is the solution of:

\[
\begin{align*}
\text{Max } u(w) + \sum_i \sum_j m_{ij} \{(1-b_{ij}) u(w_{ij}) + b_{ij} u(w_p(d^j))\}, \\
\text{s.t. (i) } \sum_i \sum_j m_{ij} [y_j - w + (1 - b_{ij}) (z(p_i^j) - w_{ij})] \geq 0, \\
& \quad (ii) \ w_{ij} \geq w_p(d^j), \ i = 1, 2, \ j = 1,..., n, \\
& \quad (iii) \ b_{ij} \in [\mu, 1],
\end{align*}
\]

where (i) implies non-negative profits and (ii) is used to indicate that the manager will quit whenever current firm wages are below his market value. Finally, (iii) is necessary to place b_{ij} in the meaningful interval. Note that the contract allows for firings through the use of b_{ij}.

We have an equilibrium, in this context, if \( \delta_p \) is the solution of (P3) given d^i, and, using the optimal b_{ij} contained in the contract, the market update coincides with d^i. The following proposition characterizes the equilibrium.

**Proposition 4:**

Whenever investment takes place in the first period, and the observed output is y_i, we have

(a) \( w_{ij} = w_{ij} = \max \{w, w_p(t^j)\}, j = 1,..., n; \)

(b) \( b_{ij} = 1 \) for all j, i.e. high-ability types are never fired;

(c) For each j, there exist w_{+}^j and w_{-}^j with w_{+}^j > w_{-}^j > w_p(t^j) such that:
c1. If $w \leq w^{-1}_j$, then $b_{ij} = 1$ and $d^i = t^i$.

c2. If $w \geq w^+_j$, then $b_{ij} = \mu$ and $d^i = q^i$.

c3. Otherwise, $b_{ij} \in (\mu, 1)$ and $d^i$ is the solution of
\[ u(w_p(d^i)) = u(w) + u'(w) (z(p_i^j) - w). \]

Proof: (See Ricart i Costa [1986]).

The interpretation of the equilibrium is very simple. Ex ante, the manager is interested in relaxing market constraints (ii) in (P3). The relaxation allows him to obtain a more uniform consumption stream. To do so, he agrees with the firm to accept being fired under certain conditions. Specifically, whenever he is of the low type and his market offer is not much below his current wage, he is fired. On the other hand, because he is risk-averse, the manager wants to insure the worst cases. This is done by assuring the manager’s employment ($b_{ij} = \mu$) whenever his pooling wage for $j$ is too low. In between both cases, there is a small interval where probabilistic firings are applied. This is controlled by $w^+_j$ and $w^+_j$. Finally, since the manager with a high pooling offer is always fired when he is of the low type, the only relevant market wage being paid by the firm is $w_p(t^i)$. That is, whenever the current employer increases wages, all the low types are fired and only the high types remain in the firm. Note that since the manager being fired may suffer a wage decrease, we have involuntary firings (ex post).

A comparison to the symmetric case is not so clear but I do have the following limiting result:

Corollary 3: As the probabilistic turnover rate $\mu$ decreases to zero, the pooling wage $w_p(t^i)$ decreases to $z(p_i^j)$ and the asymmetric situation becomes ex ante preferred by the manager to the symmetric situation.

Proof: By the definition of $w_p()$ and $t^i$, it is clear that as $\mu$ decreases to zero, $w_p(t^i)$ decreases to $z(p_i^j)$ because $t^i$ goes towards $(1, 0)$. In the limiting case, the symmetric solution is feasible for (P3). Therefore our solution is ex ante preferred by the manager.

Q.E.D.

3.4 Some intermediate cases

The separable solution presented in section 3.2 was contingent on the assumption that while wage contracts were observable, actual wages were only observable after payments were made. This means that the market can observe the contingent contract but, since it cannot observe the history on which it is contingent, it cannot figure out actual wages until payments are made at the end of the period.

It is clear from the paragraph above how important it is to ascertain exactly the information that each agent is able to observe. In what follows, I study alternative solutions depending on the observability of contracts and wages.

One extreme case corresponds to supposing that contracts cannot be observed. In this case, no contract can be used in the bargaining process to obtain an improvement from an alternative firm. In this case, the pooling solution is not unstable in the way explained in Section 3.2. Since the pooling offer cannot be used by the high-type manager to obtain a better contract, it is possible that the pooling contract will prevail in the market place. In fact, we expect that either our pooling or our separable solution will prevail, depending on which arrangement is preferred to the high-type manager. However, if only contracts, but not actual, still unpaid,
wages, are observable, then the separable solution will prevail, as discussed above, assuming that the manager is able to exploit his information.

Finally, let us assume that both contracts and wages are observable. Therefore, by observing \( w_2(h_1) \), the market can invert this function and find out about \( h_1 \). In this case, the only solution that can prevail, supposing that the current firm uses its superior information, will be the solution for the symmetric information case, because the information gets revealed through the wages. One can still think that the firm can avoid using its information, but this is infeasible as long as the manager is able to exploit his information making pooling contracts unstable.

All the cases above rely on the manager’s capacity to exploit his information. Otherwise, the solution that will prevail will be the pooling solution of section 3.3.

In summary, the pooling solution will prevail in only two extreme cases: whenever the manager is unable to exploit his information or whenever contracts are unobservable. In any other case, some amount of separation will prevail. Depending on the difficulties that the manager encounters to separate himself from lower types, the firm will be able to collect positive quasirents in the second period, thus using information as a firm-specific human capital.

4. Unverifiable Reports

In this section I want to study the contractual solution for our model when the manager’s reports are unverifiable. To illustrate the kind of problem at hand when there is no contracting, consider the example introduced in Section 3, except that \( s(t) \) can have values -1, 0, +1 with equal probability. Recall that the output is given by \( y(t) = s(t) + e(t) \), where \( e(t) \) can be 0 or 1 with \( p(e(t) = 1) = \eta \), and \( \eta \) is a variable in \([0, 1]\) that quantifies the manager’s talent. Furthermore, assume that the manager is risk-neutral and the signals are not verifiable by the owner.

In this case, the manager’s value depends on the decision rule forecast by the owner. To center the discussion, assume that the owner forecasts that the manager will use the optimal decision rule, i.e., he will invest if and only if \( s(1) = 0 \).

Since, as before, there are no incentive problems in the last period, it is easy to see that the manager’s value in the second period is:

\[
z(p(2)) = (1 + 2m_2)/3,
\]

where \( m_t \) is the expected value of \( \eta \) given \( p(t) \). To be more concrete, assume that the prior probability \( p^{(0)} \) is a Beta distribution with parameters \( \alpha \) and \( \beta \). Since the Beta and the uniform distributions are conjugated, one can easily calculate \( m_2 \) (contingent on the output) whenever the investment is accepted following the optimal rule:

\[
m_+ = (\alpha_1 + 1)/((\alpha_1 + \beta_1 + 1)) \quad \text{if } y(1) = +2
\]

\[
m_2 = m_1 \quad \text{if } y(1) = +1
\]

\[
m_- = \alpha_1/((\alpha_1 + \beta_1 + 1)) \quad \text{if } y(1) = 0.
\]

After observing \( s(1) = 0 \), if the manager presents the investment, his output can either be 1 or 0 depending on the term \( e(t) \). Therefore, his expected second-period value, given the firm’s information, is
\[ E[z^{(2)} | s(1) = 0] = 1/3 + (2/3)E[\eta m_1 + (1 - \eta)m.] < (1 + 2m_1)/3 = z^{(1)} \]

since \( m. < m_1 \). Hence, even a risk neutral manager is better off not investing when \( s(1) = 0 \).

The reason behind this result is that when the firm observes \( y(1) = +1 \), it is unable to distinguish between \( s(1) = 0 \) or \( s(1) = 1 \). This ambiguity is unfavorable to the manager.

The equilibrium lies in investing in the first period if and only if \( s(1) = +1 \). The corresponding equilibrium wages are

\[ w_1 = (1 + m_1)/3 \]
\[ w_2 = (1 + 2m_2)/3 \text{ with } E[w_2] = (1 + 2m_1)/3. \]

As a conclusion, even a risk-neutral manager acts as if he were risk-averse due to the unverifiability of the reports.

In this section, I will consider \( m \) different signals, \( m < \infty \), but I need some additional assumptions. First, I assume that the manager is paid at the beginning of the period before he is able to observe the signal. This assumption makes the wages contingent on past history, not on current performance. Second, and more important, I assume that the market value function, \( x_{ij} \), \( i = 1,..., m, j = 1,..., n \), is given. How our new results can be tied in with the previous section will be discussed later; for now, the market value of the manager in the second period is given exogenously. I will present some general results for any finite number of signals, while concentrating mainly on the two-signal case for a more complete characterization. For simplicity, I also assume the discount factor \( \beta \) is 1.

### 4.1 Some General Results

The unverifiability of signals further restricts the set of feasible contracts. By application of the revelation principle, we are restricted to the set of incentive-compatible mechanisms; therefore, assume that the owner asks the manager about the value of the signal. The contract is designed in such a way that the manager will tell the truth. A contract in this context is \( \delta = \{w, \alpha\} \) where \( w = (w, w_{ij}, v_i), i = 1,..., m, j = 1,..., n \), is the wage contract. The manager is paid \( w \) in the first period. In the second period, he is either paid \( w_{ij} \) if he invested in period one, revealed \( s_i \) and obtained \( y_j \); or \( v_i \) if he revealed \( s_i \) and the investment was rejected. And \( \alpha = (\alpha_1, ..., \alpha_m) \) is the first-period decision rule. To unify the notation denote \( z_{ij} = z(p_{ij}) \) and \( z = z(p) \). It follows that the optimal contract will be the solution to the following program:

Max \( u(w) + \sum_i r_i \left[ \alpha_i \sum_j m_{ij} u(w_{ij}) + (1-\alpha_i) u(v_i) \right] \),

s.t. (i) \( \sum_i \alpha_i \sum_j (y_j - w + z_{ij} - w_{ij}) + (1-\alpha_i) (-w+z-v_k) \geq 0 \),

(ii) \( w_{ij} \geq x_{ij}, j = 1,..., n, i \in I_+ = \{i: \alpha_i = 1\} \),

(iii) \( v_i \geq z, i \in I_+ = \{i: \alpha_i = 0\} \),

(iv) \( \alpha_i \sum_j m_{ij} u(w_{ij}) + (1-\alpha_i) u(v_i) \geq \alpha_k \sum_j m_{ij} u(w_{ij}) + (1-\alpha_k) u(v_k), \)

\( k \neq i, k,i = 1,..., m, \)

where (i) is the non-negative profit constraint. The market constraints assuring that the manager is not bid away in the second period are given in (ii) and (iii). Recall that for the time
being, we assume that nothing is learned by anyone if an investment is not made. Therefore, the manager’s reputation \( p \) is unchanged and the market is aware of it. Finally, (iv) is the incentive constraint that assures the truthful revelation of the observed signal. Let \( \gamma, \lambda_{ij}, \zeta_i, \) and \( \mu_k \) be the corresponding multipliers for (i)-(iv). After premultiplying each restriction by the unconditional probability, the Kuhn-Tucker conditions for (P4) can be written for a given decision rule \( \alpha \) as:

\[
\gamma = u'(w) \tag{13}
\]

\[
(\gamma - \lambda_{ij})/u'(w_{ij}) = 1 + \Sigma_k (\mu_k - (r_k m_{ij} \mu_{kl})/(r_l m_{ij})) \quad \text{if } i \in I_+ \tag{14}
\]

\[
(\gamma - \zeta_i)/u'(v_i) = 1 + \Sigma_k (\mu_k - (r_k \mu_{ki})/(r_i)) \quad \text{if } i \in I_- \tag{15}
\]

Define \( \tau_{ij} \) and \( \rho_i \) as the corresponding wages that solve (14) and (15) when \( \lambda_{ij} = 0 \) or \( \zeta_i = 0 \), respectively. Then it is easy to check the following partial characterization.

**Proposition 5:** The optimal wage policy for a fixed decision rule \( \alpha \) is characterized by:

(a) \( v_i = \max\{\rho_i, z\}, \quad i \in I_- \)

(b) \( w_{ij} = \max\{\tau_{ij}, x_{ij}\}, j = 1, \ldots, n, \quad i \in I_+ \)

where \( \tau_{ij} \) and \( \rho_i \) are given by

\[
1/u' (\rho_i) = (1/u'(w)) \left[ 1 + \Sigma_k (\mu_k - (r_k \mu_{kl})/(r_l)) \right] \quad \text{(16)}
\]

\[
1/u' (\tau_{ij}) = 1/u'(\rho_i) + \Sigma_k \mu_k r_k (m_{ij} - m_{ij})/(r_i u'(w)) \quad \text{(17)}
\]

(c) The market constraint multipliers are

\[
\lambda_{ij} = (u'(w)/u'(\tau_{ij})) [u'(w_{ij}) - u'(\tau_{ij})], \quad j = 1, \ldots, n, \quad i \in I_+ \tag{18}
\]

\[
\zeta_i = (u'(w)/u'(\rho_i)) [u'(v_i) - u'(\rho_i)], \quad i \in I_- \tag{19}
\]

Finally, \( \mu_k \) is given by the binding incentive constraints whenever \( \mu_k > 0 \), and \( w \) is obtained from the zero-profit condition (P4(i)).

The proof involves simple algebraic manipulations of (13)-(17) and is omitted. Obviously, the characterization is far from being an explicit solution and it may be very difficult to solve. Still, the implications stated so far are enough to extract some of the relevant properties of the optimal wage contract. To do so, let \( W, T, X \) represent the random variables (with respect to the first-period history) corresponding to the wages, the insured value \( \tau \) or \( \zeta \), and the market value \( x \), respectively. Note that the wages can be written as

\[
W = \max\{T, X\} \tag{20}
\]

Finally, let \( E[\cdot] \) represent the corresponding expected value operator. Then, for a fixed decision rule \( \alpha \), I obtain the following properties:

**Proposition 6:**

(a) \( E \left[ 1/u'(T) \right] = 1/u'(w) \);

(b) \( E \left[ 1/ut(W) \right] \geq 1/u'(w) \);

(c) if \( w_{ij} > \tau_{ij} \) then \( \lambda_{ij} > 0, \quad j = 1, \ldots, n, \quad i \in I_+ \), and if \( v_i > \rho_i \) then \( \zeta_i > 0, \quad i \in I_- \);
(d) \( v_i = v_k \) for all \( i, k \in I \).

**Proof:**

(a) \[
\mathbb{E}[1/u'(T)] = \sum \alpha_i \sum m_{ij} / u'(\tau_{ij}) + (1 - \alpha_i) / u'(\rho_i)
\]

\[
= \sum r_i / u'(\rho_i) + \sum (r_i \alpha_i / u'(w)) \sum \mu_{ik} (m_{ij} - m_{kj}) r_k / r_i
\]

\[
= \sum r_i / u'(\rho_i)
\]

\[
= (1/u'(w)) [1 + \sum k (\mu_{ik} r_k - r_k \mu_{ki})]
\]

\[
= 1/u'(w) \text{ (by telescopic sum)}.
\]

(b) Trivial because of (a), \( W \geq T \), and \( 1/u'() \). increasing.

(c) Obtained from (18)-(19).

(d) If \( \alpha_k = \alpha_i = 1 \), then from (P4(iv)) we obtain \( u(v_k) \geq u(v_i) \) and \( u(v_i) \geq u(v_k) \). Therefore, \( v_k = v_i \).

Q.E.D.

Property (d) is just a trivial observation. At present, whenever no investment is made, the signal does not reveal anything. Thus all signals \( s_i \) with \( \alpha_i = 0 \) must be treated in the same way. This observation depends on the fact that the signal alone does not reveal anything about the manager’s talent.

Properties (a) to (c) are more interesting. Property (a) is the same necessary condition obtained by Rogerson [1985] for the repeated moral hazard problem when contracts are enforceable. Note that if we assume enforceability, i.e., indenture is allowed, our solution will be \( W = T \) and we obtain exactly Rogerson’s condition. Hence the unverifiability of the signal acts in a similar way to the unobservability of an action in the standard moral hazard model.

In the corresponding verifiable version of the model, with indenture prohibited but with a public signal, one obtains the same kind of solution but with \( T = w \), which can be written as:

\[
1/u'(\tau_{ij}) = 1/u'(w), \ i = 1,...,m, \ j = 1,..., n. \tag{21}
\]

Of course, this is not true for every history in the unverifiable case. But the point of property (a) is that (21) will hold in expectation. The intuition is that the variability in the wage policy, which is necessary to induce truthful revelation, does not allow us to equalize intertemporal marginal rates of substitution for each history. Instead, we make them equal in expectation.

This point will have additional consequences. Going back to the verifiable case, the contract was downward rigid, i.e., \( W \geq w \), for all histories. Now, I obtain instead property (b), i.e., the expected marginal rate of substitution is at least one. I call this kind of contract marginally downward rigid since downward rigidity is conserved in terms of the expected marginal rate of substitution.

In the case where the signal was public, we also obtained that wages were tight, i.e., wages rose above \( w \) only when the market constraint was binding. For the unverifiable case we end up with property (c). I refer to such contracts as marginally tight. Wages rise above the insured value (with expectation satisfying (a)) only when the market constraint is binding (\( \lambda_{ij} > 0 \)).
In summary, the optimal contract is neither downward rigid nor tight. However, it is marginally downward rigid and tight. The effect of the incentive constraints is to switch from requiring that the intertemporal rates of substitution be equal to 1 for all histories, to requiring them to be unity in expectation. And the wage is above the prescribed value $T$ only when the market constraint is binding.

Furthermore, these properties will extend to a model with more periods. One insight that emerges when taking many periods into consideration is that a binding incentive constraint in one period affects the wages of all remaining periods; so that not only current reputation, but also all past history is relevant in the wage determination process (See Rogerson [1985]). This is one reason why a multiperiod characterization is extremely difficult.

For the sake of completeness, I present a corollary that follows directly from Jensen’s inequality and property (b), (Rogerson [1985]).

**Corollary 4:** If $1/u'(x)$ is a concave function of $x$, then $E[W] \geq w$.

### 4.2 Two-Signal Model

In order to understand better the characterization given before, I examine next a full characterization for the two-signal case.

**Proposition 7:** For a fixed decision rule $\epsilon_l$, the two-signal characterization is

(a) $v_i = \max\{\rho_i, z\} \quad i \in I.$

(b) $W_{ij} = \max\{\tau_{ij}, x_{ij}\} \quad i \in I^+, j = 1, ..., n$

$1/u'(\tau_{ij}) = 1/u'(\rho_i) + \mu_{ki} r_k (m_{ij} - m_{kj})/(r_i m_{ij} u'(w)), \quad i \in I^+, k \neq i, \quad j = 1,...,n,$

where $\mu_{ki} = r_i u'(w) \max\{0, 1/u'(\rho_k) - 1/u'(\rho_i)\},$

$\rho_1, \rho_2$ satisfy $r_j/u'(\rho_j) + r_j/u'(\rho_j) = 1/u'(w), \quad (22)$

and either

i. $\rho_1 = \rho_2 = w$ and no incentive constraint is binding, in which case $\tau_{ij} = p_i = w, \quad i = 1, 2$;

or

ii. If $\rho_1 > (<) \rho_2$ and then type 1 (2) jeopardizes the other type and the corresponding incentive constraint is binding (allowing us to calculate $\rho_i$'s as a function of $w$ using (22)).

Finally, $w$ is obtained from the zero-profit condition (P4(i)).

**Remark:** The only new observations in Proposition 7 are:

Equation (22) which is property (a) in Proposition 6.

The definition of $\mu_{ki}$, which is easily figured out to be a solution of (16) for $i = 1, 2$. The advantage of having only two signals is that one can solve the system of two unknowns with two equations.
4.3 The Optimal Decision Rule

I have enough tools to approach the optimal first-period decision rule. Before presenting two alternative characterizations I define what Myerson [1983] calls virtual probability distribution by

\[ M_{ij} = \frac{u'(\rho_i)}{u'(\tau_{ij})} m_{ij} \geq 0 \]  

(23)

Note that it is in fact a proper distribution since

\[ \sum_j M_{ij} = u'(\rho_i) \sum_j \frac{m_{ij}}{u'(\tau_{ij})} = u'(\rho_i) \frac{1}{u'(\rho_i)} = 1. \]

The virtual probabilities will end up being those used to calculate expected utilities in the optimal decision rule as the following proposition shows.

**Proposition 8:** \( \alpha_i = 1 \) if and only if

\[ \sum_j M_{ij} u(w_{ij}) + u'(\rho_i) \sum_j m_{ij}(y_j + z_{ij} - w_{ij}) \geq u(v_i) + u'(\rho_i) (z - v_i). \]  

(24)

**Proof:** The decision rule \( \alpha \) must maximize the Lagrangian of (P4). Then \( \alpha_i = 1 \) if and only if

\[ r_i (\sum_j m_{ij} u(w_{ij}) - u(v_i)) [1 + \sum_k (\mu_k - \mu_i r_k m_{kj}/r_i m_{ij})] + u'(w) \sum_j m_{ij} (y_j + z_{ij} - w_{ij}) - z + v_i \geq 0. \]

By rearranging,

\[ \sum_j m_{ij} u(w_{ij}) [1 + \sum_k (\mu_k - \mu_i r_k m_{kj}/r_i m_{ij})] - u(v_i) [1 + \sum_k (\mu_k - \mu_i r_k m_{kj}/r_i)] + u'(w) \sum_j m_{ij} (y_j + z_{ij} - w_{ij}) - z + v_i \geq 0. \]

Using (16) and (17) yields \( \sum_j m_{ij} (y_j + z_{ij} - w_{ij}) - z + v_i + (1/ u'(\rho_i)) \sum_i M_{ij} (u(w_{ij}) - u'(\tau_{ij})w_{ij}) - (u(v_i) - u'(\rho_i)v_i) \geq 0, \)

which after rearranging again transforms to (24). Q.E.D.

The interest in (24) lies in the use of virtual probabilities. The firm acts to maximize the manager’s virtual expected utility subject to the zero-profits constraint and with \( u'(\rho_i) \) as a multiplier. Hence, the unverifiability transforms expected utilities to virtual expected utilities (see Myerson [1983]).

To compare the unverifiable with the verifiable case, I can rearrange (24) to obtain a characterization similar to Proposition 1.

**Corollary 5:** \( \alpha_i = 1 \) if and only if \( Y(s_i, p) + L(s_i, p) + H'(s_i, p) \geq 0 \), where

\[ Y(s_i, p) = \sum_j m_{ij}y_i, \]  

(25)

\[ L(s_i, p) = \sum_j m_{ij}z_{ij} - z, \]  

(26)

\[ H'(s_i, p) = (1/ u'(\rho_i))[\sum_i M_{ij} (u(w_{ij}) - u'(\tau_{ij})w_{ij}) - (u(v_i) - u'(\rho_i)v_i)]. \]  

(27)

First note that the two first terms (25) and (26) are exactly the same as in the verifiable case studied in Proposition 1. They are, respectively, the return to financial capital and the return to learning. Also, we have a risk premium term similar (but not equal) to the one obtained before. Define

\[ g(w_{ij}) = u(x) - u'(w_i) x. \]
From HRC, the risk premium term in Proposition 2 for $\beta = 1$ is:

$$H(w_1, s_i) = E\left[ g(w_1, w_2(h_1)) \mid s_i \right]/ u'(w_1) - g(w_1, z(p(1))/u'(w_1)), \quad (28)$$

Therefore, our risk premium term must be

$$H^*(s_i, p) = E\left[ g(T, W) / u'(T) \mid s_i \right] - g(\tau, v_i)/u'(\tau). \quad (29)$$

Besides some notational changes, there are only minor differences between (28) and (29). First, we have $v_i$ in place of $z(p)$ ($p^{(1)} = p$). In fact, this is not a change since $v_i = z(p)$ in the verifiable case. We cannot be sure of this in the unverifiable case since $p_i$ may be greater than $w$. In any case, I can write (28) with $v_i$ instead of $z(p)$ to compare them more easily. Thus, the only change is due to property (a): Since our solution is only marginally downward rigid, we have a state-dependent insured value $T$, and we must use it instead of first-period wage $w_1$. In making this transformation, we go from $H$ to $H^*$.

In conclusion, the same interpretation given previously for the decision rule also applies here.

5. Extensions

We need to examine some extensions which can be easily introduced in the unverifiable reports model of the previous section. First, one can check that setting the discount factor equal to 1 is a mere convenience. We can also introduce an additional vector of observables (public information) as we did in HRC. These observable values can be used in updating beliefs about the manager’s ability. No essential results will change with this modification.

The model I have studied is only able to represent the productive aspects of a manager’s ability. It is clear that at least some of a manager’s activity is related to forecasting investment opportunities. It would be helpful to include this forecasting aspect into the model. As the model stands, only limited cases of forecasting can be studied, namely, those where $\Pr(s_i \mid \eta)$ is independent of $\eta$. We were unable to extend these cases in HRC because of our initial restriction of maintaining symmetry in beliefs. But now that the model leads to asymmetric beliefs in any case, this extension is possible.

I want to point out that, by its own nature, forecasting ability requires signals to be unverifiable. The interpretation I make of the model is as follows. The manager observes some information (which may be verifiable) and uses it to generate a forecast (the signal $s(t)$). This forecast is not verifiable since the manager must use his expertise to generate it. And, by its own nature, this process is not verifiable either. Therefore, we were unable to introduce forecasting until we had a model with unverifiable signals.

In order to consider forecasting ability in our model, I have to allow the signal $s(t)$ to be dependent on ability. I argue here that our qualitative results are unchanged by this modification. To see this, note that I can interpret $r_i$ as $\Pr(s_i \mid p)$, i.e., the probability of the signal $s_i$, given our prior beliefs, $p$, (common knowledge information) on the manager’s ability. Recall also that the second-period wages are contingent on the whole first-period history ($s, y, \phi$) where $\phi$ is a vector of additional observables. The firm, therefore, needs to update probabilities only at the end of the period, when it has gathered all the data ($s, y, \phi$). Any intermediate update is irrelevant for the firm. Obviously, the manager does update his beliefs after observing $s$, but this is already reflected
in the definition of \( m(p) \). As long as the investment is accepted, then, the model can also handle the forecasting ability interpretation.

We do have a slight problem with \( v \) when the investment is rejected. If the signal reveals something about the manager’s talent, and as long as \( v \) is independent of \( s \) (property (d)), the manager may not tell the truth when revealing a non-investment signal, and this will create asymmetric beliefs between the manager and the firm. But with some additional observable values, \( x \), and by making \( v \) contingent on \( \phi \), I can obtain a truthful revelation and avoid the whole problem.

In summary, our model can be easily adapted to account also for forecasting ability, which greatly increases the scope of our results.

I will finish this section by addressing the issue of market value, which was taken as exogenous here, but was endogenously derived in the previous model. To do so, I must specify what information is available to the market.

One possible case is where the manager’s report is observed by the market (but, of course, unverifiable). Since the firm is using an incentive-compatible contract, the report is as good as the signal and the market can infer perfectly each manager’s type. In this case, I have \( x_{ij} = z_{ij} \) whenever \( \alpha_i = 1 \). (Note that this will imply that \( v_k = z \) for all \( k \) such that \( \alpha_k = 0 \), as we had in our verifiable case).

Alternatively, one can consider the case where the market can neither observe the manager’s reports nor the signal. Therefore, the investment decision and the output are the only information received by the market. This is the situation in Section 3. As I argued there, if the manager can signal his type through risk sharing or any other mechanism, I expect to see some kind of discriminating market value function where low types are paid their marginal product, and high types are paid less than their marginal product due to the cost of communicating information. Whenever discrimination is not possible, for instance, when the manager is paid at the beginning of the period, the market will have to offer a pooling contract. In this case, as I discussed before, the firm’s incentive to fire low types and retain high types will bring the pooled market value close to the marginal product of low types.

The main point is that even though I studied the two asymmetries in separate models, the main qualitative insights remain in a merged model.

6. Conclusions

It is important to understand which circumstances make a particular contract optimal for a particular situation. More than the specific quantitative results, it is interesting to understand the qualitative relationships and the different components of a contract.

In this paper, I have dealt with asymmetric information. For reasons of tractability only simple 0-1 investment decisions were studied but I believe that the qualitative results are much more general. I considered two types of asymmetries, the asymmetry in the labor market and the one generated by the unverifiability of the manager’s reports.
When we have asymmetries in the managerial labor market, the firm is able to collect quasi-rents from talented managers in the same way as when we consider some kind of firm-specific human capital. Furthermore, depending on the possibilities that the manager has to exploit his private information, the optimal contract may contain some amount of firings.

The effect of the unverifiability of the manager’s reports can be broken down into two parts. First, to obtain a truthful revelation, the insured wage is stochastic, and dependent on the likelihood ratio much in the same way as the standard moral hazard problem. Second, the solution is essentially the same as the symmetric case but with the stochastic wage guarantee.

Clearly, more work is needed in order to understand the effects of combined asymmetries in information. In the introduction, I tried to relate the results in this paper with the known results on incentive contracts with the objective of understanding the main components of incentive contracts and their cause. More work needs to be done. For instance, a major informational change would be to allow the manager to take discretionary actions. However, I have attempted to move forward in understanding the interrelation between information and reputation and their combined effect on contracts and decisions.
References


